

Special Cases

No Solution

When you follow the steps to solve an **equation**, you try to isolate the variable. You have a solution when you get the equation $x = \text{some value}$. There are equations, however, that have no solution, and other equations that have an infinite number of solutions. How does this work?

✓ Example

Solve for x : $12 + 2x - 8 = 7x + 5 - 5x$

Solution

$$12 + 2x - 8 = 7x + 5 - 5x \quad \text{Combine **like terms** on both sides of the equation.$$

$$\begin{array}{r} 2x + 4 = 2x + 5 \\ -2x \quad -2x \\ \hline 4 = 5 \end{array} \quad \text{Isolate the x term by subtracting $2x$ from both sides.}$$

This is *not* a solution! You did *not* find a value for x . Solving for x the way you know how, you arrive at the false statement $4 = 5$. Surely 4 cannot be equal to 5!

This may make sense when you consider the second line in the solution where like terms were combined. If you multiply a number by 2 and add 4 you would never get the same answer as when you multiply that same number by 2 and add 5. Since there is no value of x that will ever make this a true statement, the solution to the equation above is “no solution.”

Be careful that you do not confuse the solution $x = 0$ with “no solution.” The solution $x = 0$ means that the value 0 satisfies the equation, so there is a solution. “No solution” means that there is no value, not even 0, which would satisfy the equation.

Also, be careful not to make the mistake of thinking that the equation $4 = 5$ means that 4 and 5 are values for x that are solutions. If you substitute these values into the original equation, you’ll see that they do not satisfy the equation. This is because there is truly *no solution*—there are no values for x that will make the equation $12 + 2x - 8 = 7x + 5 - 5x$ true.

Special Cases

All Real Numbers (Infinite Solutions)

You have seen that if an equation has no solution, you end up with a false statement instead of a value for x . You can probably guess that there might be a way you could end up with a true statement instead of a value for x .

✓ Example

Solve for x .

$$5x + 3 - 4x = 3 + x$$

Solution

$$5x + 3 - 4x = 3 + x \quad \text{Combine like terms on both sides of the equation.$$

$$\begin{array}{r} x + 3 = 3 + x \\ -x \quad \quad -x \\ \hline 3 = 3 \end{array} \quad \text{Isolate the x term by subtracting x from both sides.}$$

You arrive at the true statement " $3 = 3$." When you end up with a true statement like this, it means that the solution to the equation is "all real numbers." Try substituting $x = 0$ into the original equation—you will get a true statement! Try $x = -\frac{3}{4}$, and it also will check!

This equation happens to have an infinite number of solutions. Any value for x that you can think of will make this equation true. When you think about the context of the problem, this makes sense—the equation $x + 3 = 3 + x$ means "some number plus 3 is equal to 3 plus that same number." We know that this is always true—it's the commutative property of addition!