Quadratic Function Application - Business

The Problem

The owners of a travel agency have determined that they can sell all 160 tickets for a tour if they charge \$8 (their cost) for each ticket. For each \$0.25 increase in the price of a ticket, they predict (based upon prior ticket sales and statistical analysis) that they will sell one less ticket. Let x represent the number of tickets sold. Find the cost function, $C(x)$, the customer's price function, $p(x)$, the profit function, $P(x)$, the maximum profit, and the cost per ticket that yields the maximum profit.

The Cost Function

This function models the total cost incurred by the travel agency for the tour.
Cost = (cost per ticket)(# of tickets sold). Since the cost per ticket is \$8 and the number of tickets sold is x, the cost function is $C(x) = 8x$.

The Customer's Price Function

This function models the price of the customers' tickets based on the prediction that they will sell one less ticket for each \$0.25 price increase. Note that if they sell 160 tickets (x is the independent variable, the number of tickets sold) they charged \$8 per ticket, and we have the ordered pair $(160,8)$. Also, if the sell 159 tickets, they charged \$8.25 per ticket, and the ordered pair is $(159,8.25)$. Now find the equation of the line passing through the two ordered pairs. The slope is:

$$\frac{8.25-8}{159-160} = \frac{0.25}{-1} = -0.25.$$

Use the point-slope equation to find the price function, where $y = p(x)$:

$$p(x) - 8 = -0.25(x-160)$$
$$p(x) - 8 = -0.25x + 40$$
$$p(x) = -0.25x + 48$$
$$p(x) = 48 - 0.25x$$

The Profit Function

The profit function is $P(x) = R(x) - C(x)$ where $R(x)$ the revenue function is and $C(x)$ is the cost function. Note that revenue = (# of tickets sold)(price per ticket). The number of tickets sold is x, and the price per ticket is $p(x)$. Thus the profit function is:

$$P(x) = R(x) - C(x)$$
$$P(x) = x\left[p(x)\right] - C(x)$$
$$P(x) = x(48 - 0.25x) - 8x$$
$$P(x) = 48x - 0.25x^2 - 8x$$
$$P(x) = 40x - 0.25x^2$$

The Maximum Profit

The maximum profit occurs at the vertex of the profit function:

$$x = -\frac{b}{2a} = -\frac{40}{2(-0.25)} = 80$$

So 80 tickets will maximize the profit. The profit will be:

$$P(80) = 40(80) - 0.25(80)^2$$
$$= 1600$$

The Cost per Ticket Yielding Maximum Profit

To find the price per ticket that yields the maximum profit, evaluate $p(x)$ with $x = 80$:

$$p(80) = 48 - 0.25(80) = 28$$

Final Analysis

So the travel agency can expect a maximum profit of \$1600 when 80 people take the tour at a ticket price of \$28 per person.

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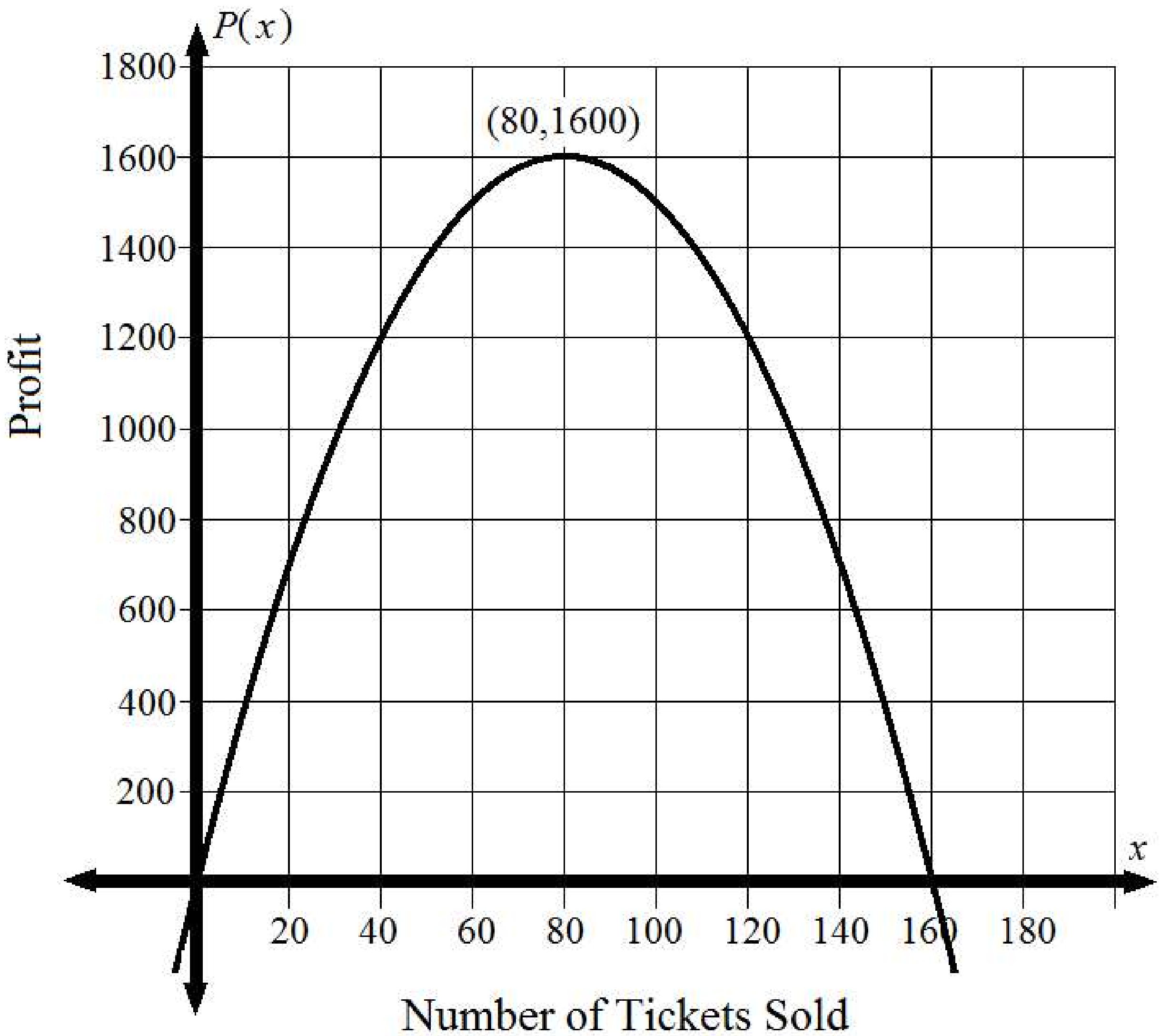
The graph of the profit function $P(x) = 40x - 0.25x^2$ **:**



The **domain** of the function based on the **graph** is $\{x \mid x \in \mathbb{R}\}$, or in interval notation, $(-\infty, +\infty)$.

The **domain** of the function based on the **context** is $\{x \mid 0 \le x \le 160\}$, or in interval notation, $[0, 160]$.

The **range** of the function based on the **graph** is $\{P(x) \mid P(x) \le 1600\}$, or in interval notation, $(-\infty, 1600]$.

The **range** of the function based on the **context** is $\{P(x) \mid 0 \le P(x) \le 1600\}$, or in interval notation, $[0, 1600]$.

The **zeros** of the function can be found by factoring $P(x) = 40x - 0.25x^2$ as $P(x) = -0.25x(x - 160)$. Using the Zero Factor Property, the zeros are $x = 0$ and $x = 160$, which are the values of x which make a profit of \$0. The travel agency must sell between 0 and 160 tickets to make a profit, or $0 < x < 160$.