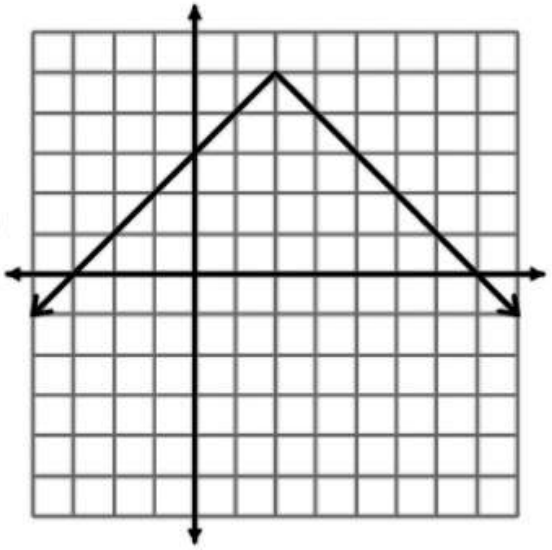
Warm-Up

Use the graph of the function y = f(x) which is shown below to answer the questions.



1.Evaluate f(3) 4

2.What values of x solve the equation f(x) = 0?

X = -3 x = 7

3.For what values of x is f(x) increasing?

(-∞, 2)

4.For what values of x is f(x) decreasing?

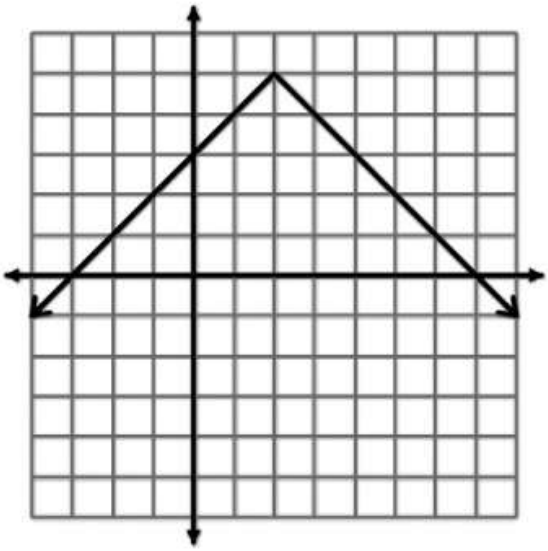
(2, ∞)

5.What is the domain and range of f(x)

D: R R: y ≤ 5

Warm-Up

Use the graph of the function y = f(x) which is shown below to answer the questions.



1.Evaluate f(3) _____

2.What values of x solve the equation f(x) = 0?

3.For what values of x is f(x) increasing?

4.For what values of x is f(x) decreasing?

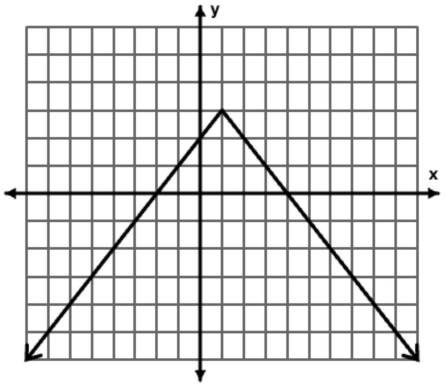
5.What is the domain and range of f(x)?

Introduction to Absolute Value Functions

Student Handout

Absolute Value Functions have the form $f(x) = a|x - c| + d$ and when graphed will form a v-shaped graph which will have a minimum value when a > 0 and will have a maximum value when a < 0.

Example 1: The function $f(x) = -|x - 1| + 3$ is shown below. Use the graph to answer the questions



a) The function is symmetric about the line x = 1

We call this line the axis of symmetry (AOS)

b) Vertex: (1, 3) c) y-intercept: (0, 2)

c) The function has a min/max value of 3 at x = 1

d) Domain: R Range: y ≤ 3

Example 2: Generate a table for the function $f(x) = |x + 2| + 3$ for the interval x = -6 to x = 2 and then answer the questions.

x	-6	-5	-4	-3	-2	-1	0	1	2
$f(x)$	7	6	5	4	3	4	5	6	7

a) The function is symmetric about the line x = -2 b) The vertex is (-2, 3)

c) y-intercept: (0, 5) d) The function has a max/min of 3 at x = -2

e) Increasing interval (3, ∞) f) Decreasing interval (-∞, 3)

g) Domain: R h) Range y ≤ 3

Example 3: Determine whether the function will have a minimum or maximum, and then find the axis of Symmetry, vertex, and y-intercept.

a) $f(x) = 2|3x - 6| + 2$

Max/Min

AOS: x = 2

Vertex: (2, 2)

y-int: (0, 14)

b) $g(x) -\frac{1}{2}|2x + 8| - 2$

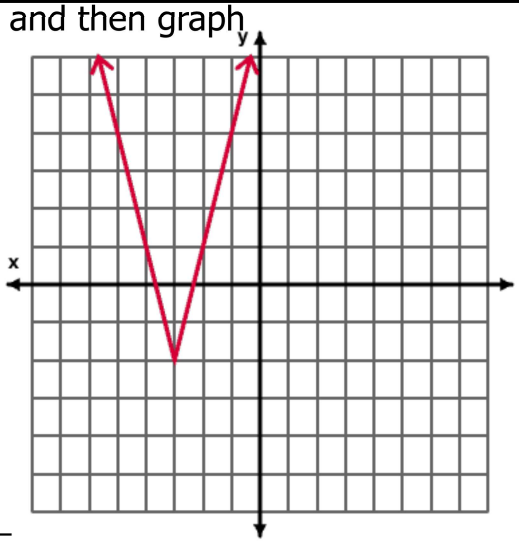
Max/Min

AOS: x = -4

Vertex: (-4, -2)

y-int: (0, -6)

Example 4: If $f(x) = 3|x + 3| - 2$, find each of the following and then graph



a) Find the axis of symmetry and plot _____x = -3_____

b) Find the vertex and plot _____(-3, -2)_____

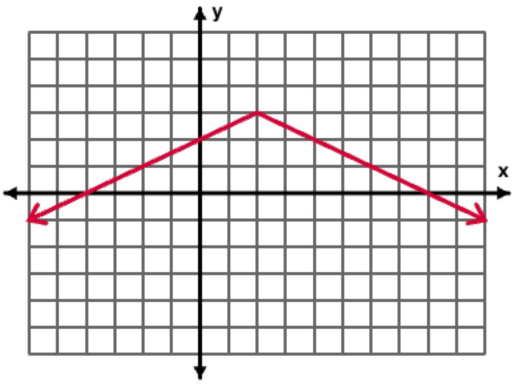
c) Use the value of "a" to plot the right side of the V-shape

A = 3

d) Use symmetry to plot the left side of the V-shape

e) The graph has a min/max value of _____-2_____ at x = _____-3_____

Example 5: If $f(x) = -\frac{1}{2}|x - 2| + 3$, find each of the following and then graph



a) Find the axis of symmetry and plot

x = 2

b) Find the vertex and plot

(2, 3)

c) Use the value of "a" to plot the right side of the V-shape

A = -1/2

d) Use symmetry to plot the left side of the V-shape

e) The graph has a min/max value of _____3_____ at x = _____2_____

Example 6: The number of shoppers in a store is modeled by the function below:

$$s(t) = -\frac{1}{2}|t - 288| + 144$$

where t is the time (in minutes) since the store opened at 10:00 A.M.

a) What is the greatest number of shoppers in the store?

Max Value: 144

b) At what time does the greatest number of shoppers occur?

288 minutes after opening at 10:00 or 2:48

Name_____ Period_____

Exit Ticket

1.Tell whether the function will have a minimum or a maximum

a) $f(x) = -2|x - 5| + 7$

Max

b) $g(x) = 3|x + 2| - 4$

Min

2. If $f(x) = |x + 5| - 2$, find each of the following

a) AOS: $x = -5$

b) Vertex: $(-5, -2)$

c) y-intercept

$(0, 3)$

d) The function has a min/max of _-2_ at x = _-5_

Name_____ Period_____

Exit Ticket

1.Tell whether the function will have a minimum or a maximum

a) $f(x) = -2|x - 5| + 7$

b) $g(x) = 3|x + 2| - 4$

2. If $f(x) = |x + 5| - 2$, find each of the following

a) AOS:

b) Vertex:

c) y-intercept

d) The function has a min/max of _____ at x = _____

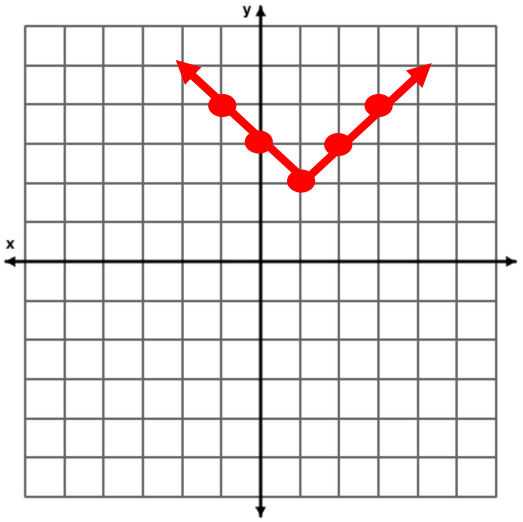
Name_____ Period_____

Practice

Introduction to Absolute Value

In 1 – 3, find the requested information and then graph

1. $f(x) = |x - 1| + 2$



AOS: x = 1

Vertex: (1, 2)

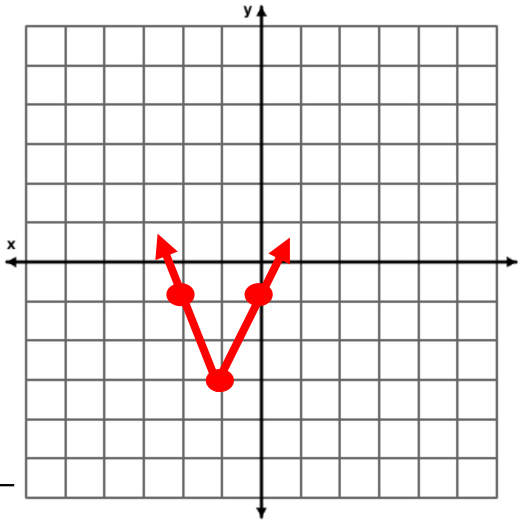
Value of a: 1

y-intercept: (0, 3)

The function has a minimum value of 2 at x = 1

Domain R f) Range: y ≥ 2

2. $f(x) = 2|x + 1| - 3$



AOS: x = -1

Vertex:: (-1. -3)

Value of a: 2

y-intercept: (0, 1)

The function has a minimum value of -3 at x = -1

Domain R f) Range: y ≥ -3

3. $f(x) = -\frac{1}{2}|x - 1| + 4$

AOS: x = 1

Vertex:: (1, 4)

Value of a: -1/2

y-intercept: (0, 3.5)

The function has a maximum value of 4 at x = 1

Domain R f) Range: y ≤ 4

4. The table below shows some values of a quadratic functions. Use the values to answer the questions.

x	0	1	2	3	4	5	6
f(x)	-1	0	1	2	1	0	-1

a) What is the axis of symmetry? x = 3 b) What is the vertex? (3, 2)

c) What is the y-intercept? (0, -1) d) What is the maximum value? 2

e) Increasing interval (-∞, 3) f) Decreasing interval (3, ∞)

g) What is the domain? R h) What is the range? y ≤ 2

5. Given $f(x) = 2|x + 1| - 3$, find each of the following

a) What is the axis of symmetry? x = -1 b) What is the vertex? (-1, -3)

c) What is the y-intercept? (0, -1) d) What is the maximum value? -3

e) Increasing interval (-1, ∞) f) Decreasing interval (-∞, -1)

g) What is the domain? R h) What is the range? y ≥ -3

5. A band's new album is released. Weekly sales, (in thousands), increase steadily for a while and then decrease according to the model $s(t) = -2|t - 22| + 44$ where t represents the time in weeks.

a) In what week did their largest sales occur? Week 22

b) What were their sales for this week? $44,000

Warm-Up

Use the graph of g(x) shown below to answer the questions.

a) g(x) is symmetric about the line **x = -1**

b) The vertex of g(x) is **(-1, 2)**

c) g(x) has a maximum value of **2** at x = **-1**

d) What are the zeros of the function?

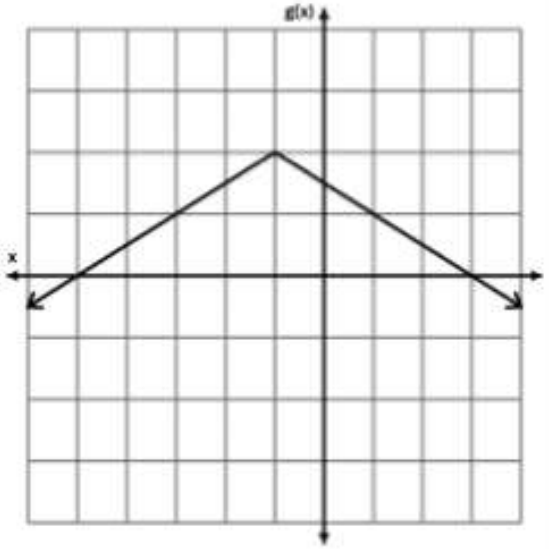
x = -5 x = 3

e) What is the domain of the function?

All Real Numbers

f) What is the range of the function?

y ≤ 2



Warm-Up

Use the graph of g(x) shown below to answer the questions.

a) g(x) is symmetric about the line _____

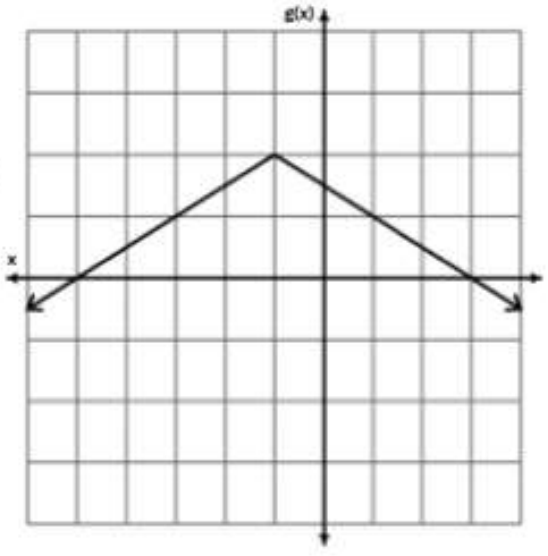
b) The vertex of g(x) is _____

c) g(x) has a maximum value of _____ at x = _____

d) What are the zeros of the function?

e) What is the domain of the function?

f) What is the range of the function?



Transformations of

Absolute Value Functions

If |a| > 1, Vertical Stretch
If 0 < |a| < 1, Vertical Compression
If a < 0, reflect across the x-axis

(x, y) → (x, ay) "a" only affect y!

f(x) + d; Vert. Shift Up
f(x) - d; Vert Shift Down

(x, y) → (x, y + d)
(x, y) → (x, y - d)

$$g(x) = a|b(x - c)| + d$$

If |b| > 1, Horizontal Compression
If 0 < |b| < 1, Horizontal Stretch
If b < 0, reflect across the y-axis
(x, y) → (x/b, y) "b" only affect x-values!

f(x + c); Horizontal shift left c units
(x, y) → (x - c, y)

f(x - c); Horizontal shift right c units
(x, y) → (x + c, y)

Transformations of Absolute Value Functions I

Student Handout

Transformations can be applied to the parent graph f(x) = |x| just like any other function by changing the values of a, b, c, and d in the function

$$f(x) = a|b(x - c)| + d$$

Example 1: If $f(x) = |x|$ and $g(x) = f(x) + 2$, determine what operations must be performed on x or y, create a table for g(x), sketch, and then describe the transformation.

To obtain g(x), _____

Add 2 to the y-values

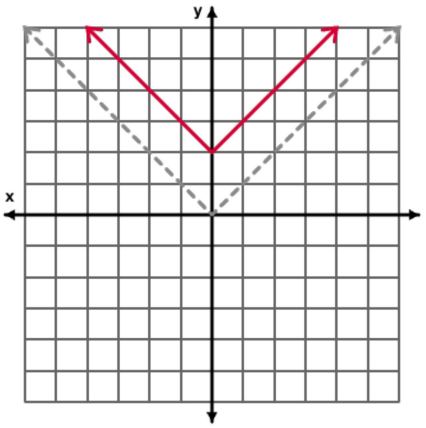
_____

Transformation:

Up 2

x	f(x)
-4	4
0	0
4	4

x	g(x)
-4	**6**
0	**2**
4	**4**



Example 2: If $f(x) = |x|$ and $g(x) = f(x - 2)$, determine what operations must be performed on x or y, create a table for g(x), sketch, and then describe the transformation.

To obtain g(x), _____

Add 2 to the x-values

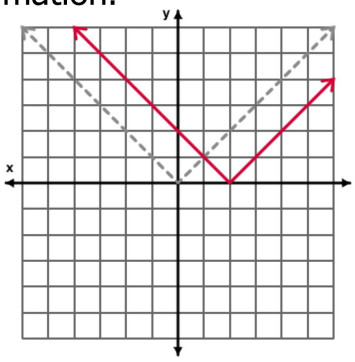
_____

Transformation:

Right 2

x	f(x)
-4	4
0	0
4	4

x	g(x)
-2	4
2	0
6	4



Example 3: If If $f(x) = |x|$ and $g(x) = -f(x)$,and $h(x) = f(-x)$, determine what operations must be performed on x or y, create a table for g(x) and h(x), sketch, and then describe the transformation.

To obtain g(x), _____

Multiply y-values by -1.

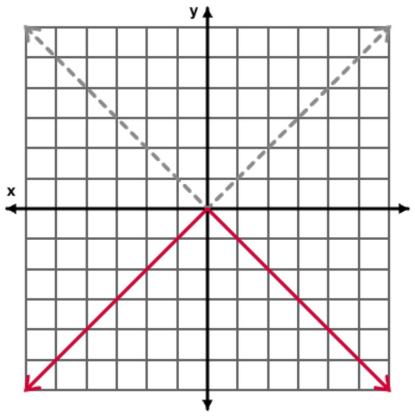
To obtain h(x):_____

Multiply x-values by -1.

x	f(x)
-4	4
0	0
4	4

x	g(x)
-4	**-4**
0	**0**
4	**-4**

x	h(x)
-4	4
0	0
4	4



Transformations:

g(x) is a reflection across x-axis

H(x) is a reflection across the y-axis

And the same as original because function is even.

Example 4: If $f(x) = |x|$ and $g(x) = \frac{1}{2}f(x)$, determine what operations must be performed on x or y, create a table for g(x), sketch, and then describe the transformation.

To obtain g(x),_____

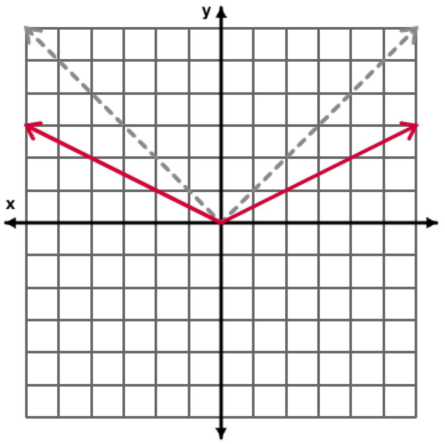
Multiply y-values by ½

Transformation:

Vertical Compression by a factor of (bfo) of ½

x	f(x)
-4	4
0	0
4	4

x	g(x)
-4	2
0	0
4	2



Example 5: If $f(x) = |x|$ and $h(x) = f(\frac{1}{2}x)$, determine what operations must be performed on x or y, create a table for g(x), sketch, and then describe the transformation.

To obtain h(x),_____

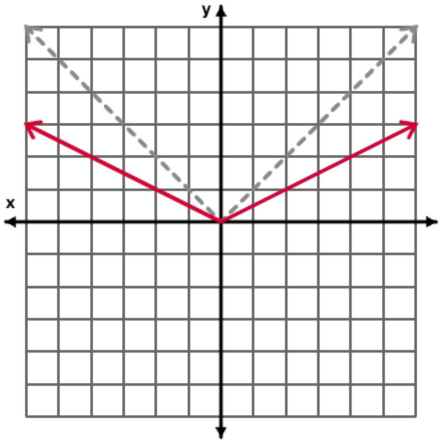
Multiply x-values by 2

Transformation:

Horizontal Stretch by a factor of 2

x	f(x)
-4	4
0	0
4	4

x	h(x)
-8	4
0	0
8	4



Example 6: An absolute value function, f, has a vertex of (6, -2). What would the new vertex be if f is transformed according to the rules below?

a) $g(x) = f(x + 5)$

(x, y) → (x – 5, y)

New Vertex:

(1, -2)

b) $g(x) = 2f(x)$

(x, y) → (x , 2y)

New Vertex:

(6, -4)

c) $g(x) = f(2x)$

(x, y) → (1/2x, y)

New Vertex:

(3, -2)

Name_____ Period_____

Exit Ticket

If g(x) is obtained from f(x), describe the operations which must be performed on x or y and then create a table of values for g(x). Finally, describe the transformation in words.

1. $f(x) = |x|$ and $g(x) = f(x-3)$

To obtain g(x), _____

Add 3 to the x-values

x	f(x)
-4	4
0	0
4	4

x	g(x)
-1	4
3	0
7	4

Describe the transformation in words:

Right 3

2. $f(x) = |x|$ and $g(x) = 2f(x)$

To obtain g(x), _____

Multiply the y-values by 2

x	f(x)
-4	4
0	0
4	4

x	g(x)
-4	**8**
0	**0**
4	**8**

Describe the transformation in words:

Vert. Stretch bfo 2

Name_____ Period_____

Exit Ticket

If g(x) is obtained from f(x), describe the operations which must be performed on x or y and then create a table of values for g(x). Finally, describe the transformation in words.

1. $f(x) = |x|$ and $g(x) = f(x-3)$

To obtain g(x), _____

x	f(x)
-4	4
0	0
4	4

x	g(x)
	4
	0
	4

Describe the transformation in words:

2. $f(x) = |x|$ and $g(x) = 2f(x)$

To obtain g(x), _____

x	f(x)
-4	4
0	0
4	4

x	g(x)
-4	
0	
4	

Describe the transformation in words:

Name_____ Date_____ Period_____

Practice Transformations of Absolute Value Functions I

For 1 - 5, describe how to find the table values for g, complete the table and sketch. Finally, describe the transformation.

1. If $f(x) = |x|$ and $g(x) = 3f(x)$,

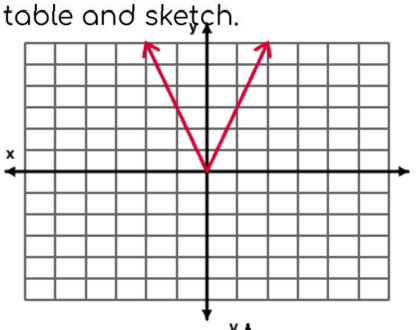
to obtain g(x), multiply the ~~y~~ values by 3

Transformation:

Vert. Stretch bfo 3

x	f(x)
-4	4
0	0
4	4

x	g(x)
-4	12
0	0
4	12



2. If $f(x) = |x|$ and $g(x) = f(x+3)$,

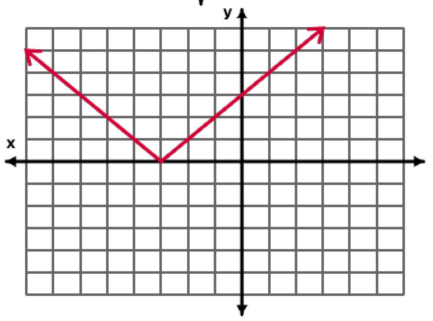
to obtain g(x), Subtract 3 from the x-values

Transformation:

Left 3

x	f(x)
-4	4
0	0
4	4

x	g(x)
-7	4
-3	0
1	4



3. If $f(x) = |x|$ and $g(x) = f(x) - 3$

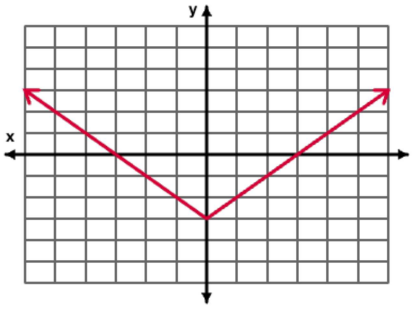
to obtain g(x), subtract 3 from the y-values.

Transformation:

Down 3

x	f(x)
-4	4
0	0
4	4

x	g(x)
-4	1
0	-3
4	1



4. If $f(x) = |x|$ and $g(x) = 2f(x)$

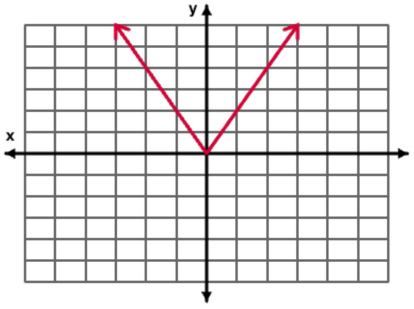
to obtain g(x), multiply the y-values by 2

Transformation:

Vert. Stretch bfo 2

x	f(x)
-4	4
0	0
4	4

x	g(x)
-4	8
0	0
4	8



5. If $f(x) = |x|$ and $g(x) = f(x-5)$

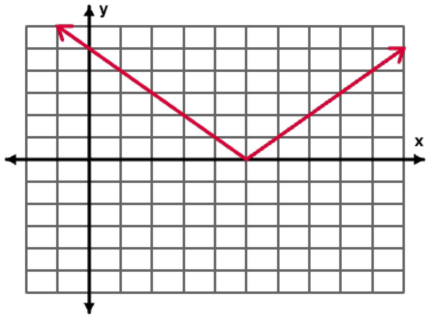
to obtain g(x), Add 5 to the x values.

Transformation:

Right 5

x	f(x)
-4	4
0	0
4	4

x	g(x)
1	4
5	0
9	4



Warm-Up

Describe the operation which must occur on x or y and then describe the transformation which occurs (up, down, left, right, etc)

1. $g(x) = 5|x|$

To obtain $g(x)$,

Multiply y-values by 5

Description of transformation:

V. Stretch bfo 5

2. $g(x) = |x - 5|$

To obtain $g(x)$,

Add 5 to the x-values

Description of transformation:

Shift right 5

3. $g(x) = |x| + 5$

To obtain $g(x)$,

Add 5 to the y-values

Description of transformation:

Shift Up 5

Warm-Up

Describe the operation which must occur on x or y and then describe the transformation which occurs (up, down, left, right, etc)

1. $g(x) = 5|x|$

To obtain $g(x)$,

Description of transformation:

2. $g(x) = |x - 5|$

To obtain $g(x)$,

Description of transformation:

3. $g(x) = |x| + 5$

To obtain $g(x)$,

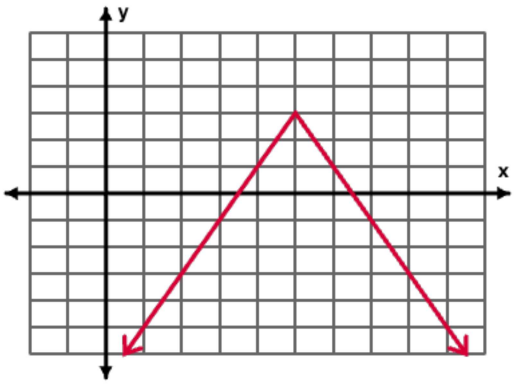
Description of transformation:

Transformations of Absolute Value Functions II

In today's lesson, we will explore how we can use the properties of transformations to sketch the graph of an absolute value equation and write an absolute value equation to describe a given graph.

Example 1: Identify the transformations and graph the function:

$$f(x) = -2|x - 5| + 3$$



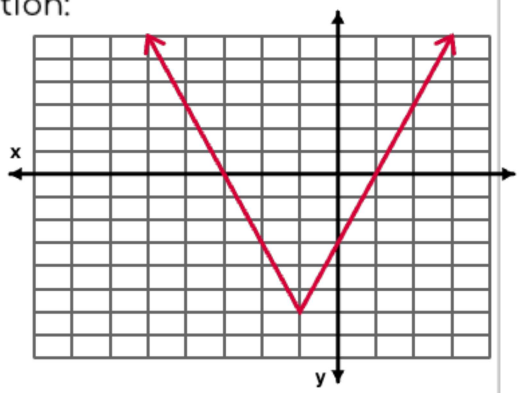
Since a = -2, the new graph will be reflected across the x axis and vertically stretched bfo 2.

Because "c" is 5, the graph will be shifted 5 units right

Because "d" is 3, the graph will be shifted 3 units Up

Example 2: Identify the transformations and graph the function:

$$g(x) = 3|x + 1| - 6$$



Transformations: a = 3; V. Stretch by factor of 3

c = -1; Left 1

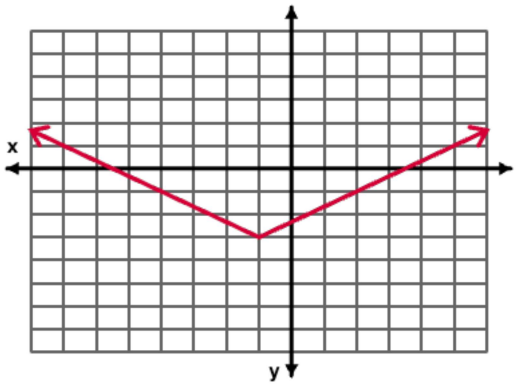
d = -6; Down 6

Vertex: (-1, -6)

Domain: All Real Numbers Range: {y | y ≥ -6}

Example 3: Identify the transformations and graph the function:

$$g(x) = \frac{2}{3}|x + 1| - 3$$



Transformations: a = 2/3; V. Comp. bfo 2/3

c = -1; Left 1

d = -3; down 3

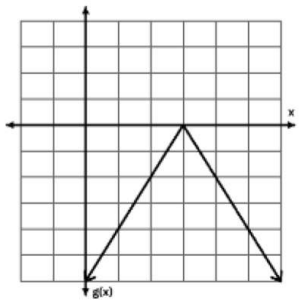
Vertex: (-1, -3)

Domain: All Real Numbers Range: {y | y ≥ -3}

Example 5: The graph of g(x) is graphed below. Describe the transformations on f(x) = |x| required to obtain g(x) with a verbal description and then write an equation to describe g(x).

a)

Reflect across x-axis
V. Stretch bfo 2
Right 3



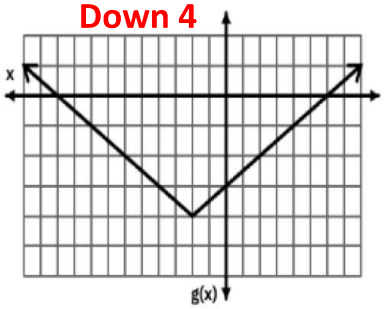
a = -2 c = 3 d = 0

Equation:

G(x) = -2|x – 3|

b)

Vert. Comp bfo ½
Left 2
Down 4



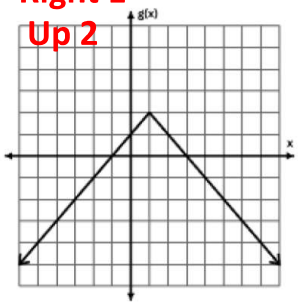
a = ½ c = -2 d = -4

Equation:

g(x) = ½ |x + 2| - 4

c)

Reflect across x-axis
Right 1
Up 2



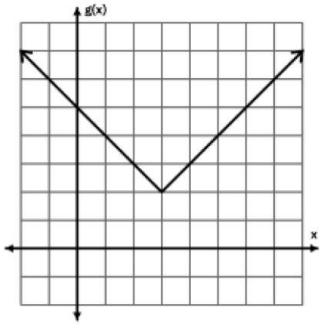
a = -1 c = 1 d = 2

Equation:

g(x) =-|x – 1| + 2

d)

Right 3
Up 2



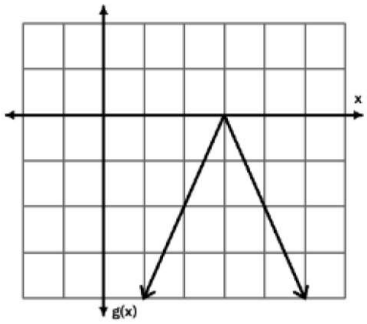
a = 1 c = 3 d = 2

Equation:

G(X) = |x – 3| + 2

e)

Reflect across x-axis
Right 3



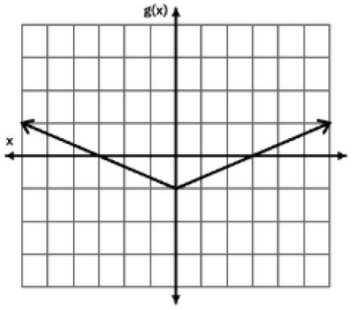
a = -2 c = 3 d = 0

Equation:

g(x) = -2|x – 3|

f)

V. Comp bfo 1/3
Down 1



a = 1/3 c = 0 d = -1

Equation:

g(x) = 1/3|x| - 1

Name_____ Date_____ Period_____

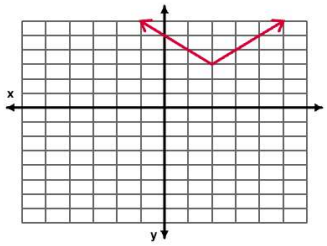
Practice Transformations of Absolute Value Functions II

Give a verbal description of the transformation that was used to transform each of the following from the function f(x) = |x|, state the vertex, and then graph.

1. $f(x) = |x - 2| + 3$

Transformations:
Rt 2, Up 3

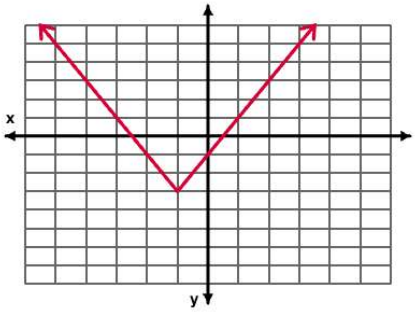
Vertex: (2, 3)



2. $f(x) = 2|x + 1| - 3$

Transformations:
V. Stretch bfo 2
Left 1; down 3

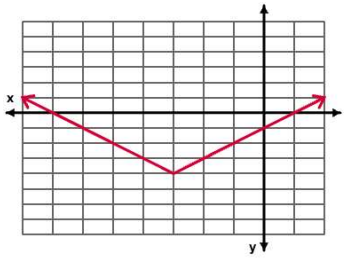
Vertex: (-1, -3)



3. $f(x) = |x + 3| - 4$

Transformations:
Left 3, down 4

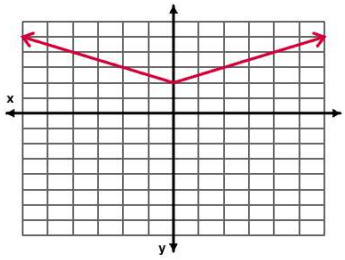
Vertex: (-3, -4)



4. $f(x) = \frac{1}{2}|x| + 2$

Transformations:
V. Comp bfo ½
Up 2

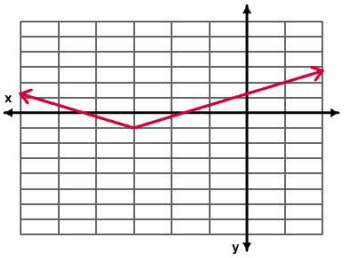
Vertex: (0, 2)



5. $f(x) = \frac{3}{4}|x + 3| - 1$

Transformations:
V. Comp bfo ¾
Left 3
Down 1

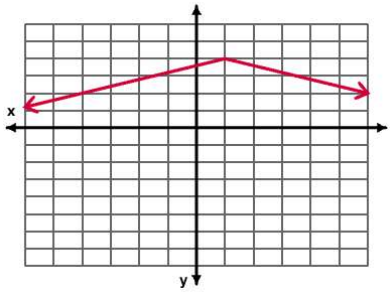
Vertex: (-3, -1)



6. $f(x) = -\frac{2}{5}|x - 1| + 4$

Transformations:
Reflect across x-axis
V.Comp bfo 2/5
Right 1
Up 4

Vertex: (1, 4)



7. An absolute value function, g(x), is obtained from f(x) = |x| by shifting f(x) down 5 and right 3. Which equation could represent g(x)?

A f(x) = |x + 3| - 5 B f(x) = 3|x| - 5 C f(x) = |x - 3| - 5

C

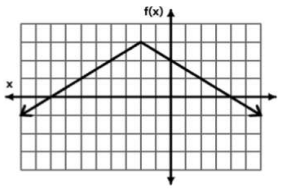
8. An absolute value function, g(x), is obtained by reflecting f(x) across the x-axis, stretching f(x) bfo 2, and shifting f(x) left 1 and up 5.

A f(x) = 2|x + 1| + 5 B f(x) = 2|-x| + 5 C f(x) = -2|x + 1| + 5

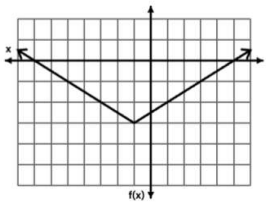
C

Match each graph with its equation

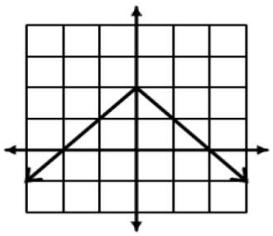
9. C.



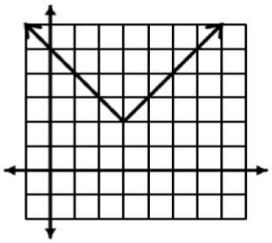
10. H



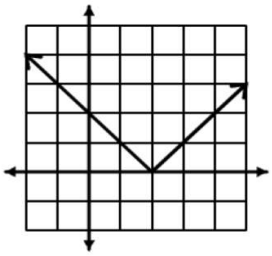
11. E



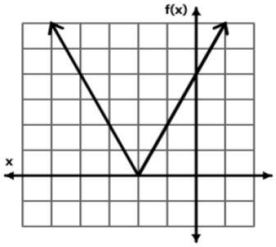
12. I



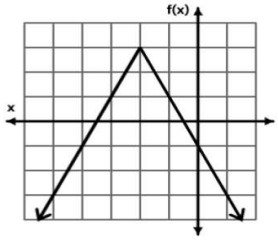
13. G



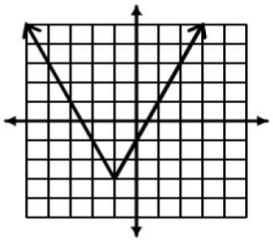
14. A



15. D



16. F



A $f(x) = 2\|x+2\|$	B $f(x) = -\|x\| + 2$	C $f(x) = -\frac{1}{2}\|x+2\| + 3$
D $f(x) = -2\|x+2\| + 3$	E $f(x) = \|x\| + 2$	F $f(x) = 2\|x+1\| - 3$
G $f(x) = \|x-2\|$	H $f(x) = \frac{1}{2}\|x+1\| - 3$	I $f(x) = \|x-3\| + 2$

Name_____ Period_____

Exit Ticket

Transformations of Absolute Value Functions II

1. The graph of g(x) is graphed below. Describe the transformations on f(x) = |x| required to obtain g(x) with a verbal description and then write an equation to describe g(x).

Verbal Description of Transformations:

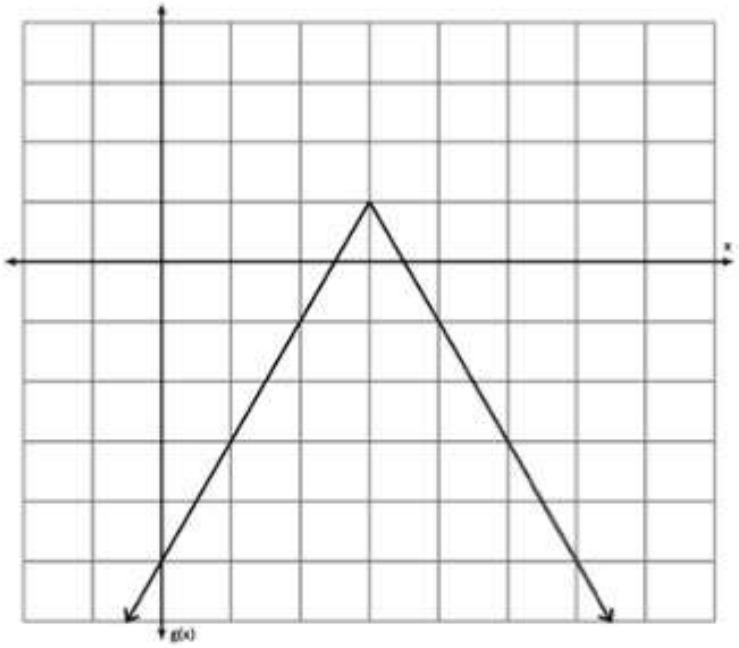
Reflect across x-axis

V. Stretch bfo 2

~~**Right 3**~~

Up 1

Equation for g(x): **g(x) = -2|x-3|+ 1**



Name_____ Period_____

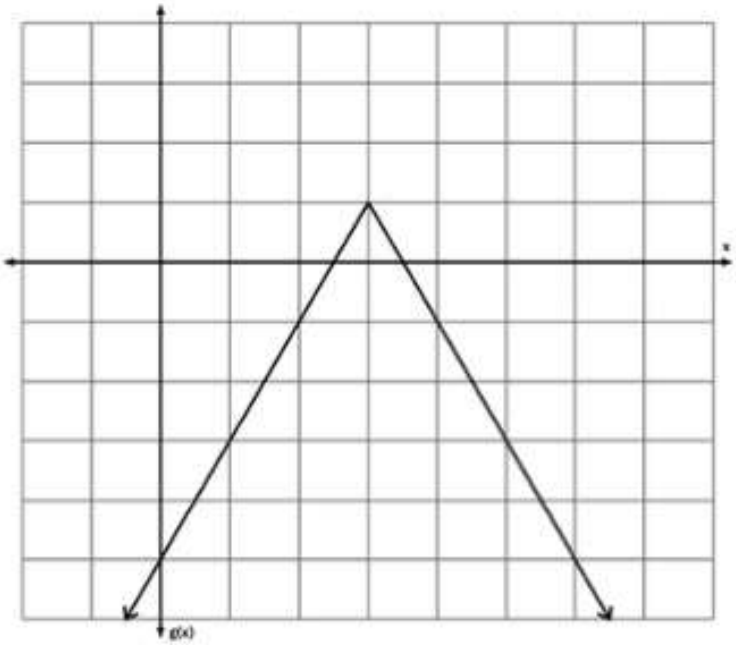
Exit Ticket

Transformations of Absolute Value Functions II

1. The graph of g(x) is graphed below. Describe the transformations on f(x) = |x| required to obtain g(x) with a verbal description and then write an equation to describe g(x).

Verbal Description of Transformations:

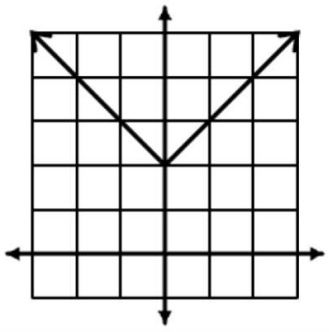
Equation for g(x): _____



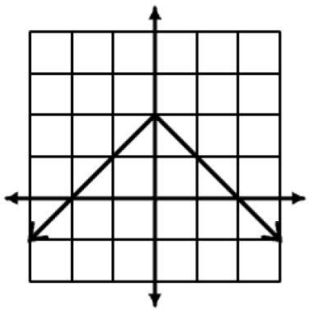
Name_____ Date_____ Period_____

Who Am I?
Transformations of Absolute Value Functions

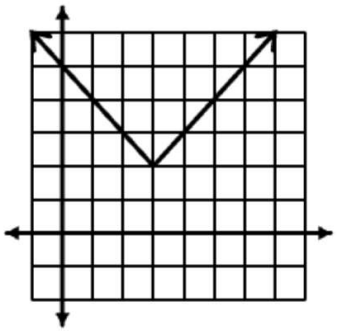
A



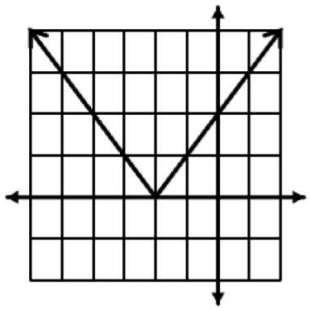
B



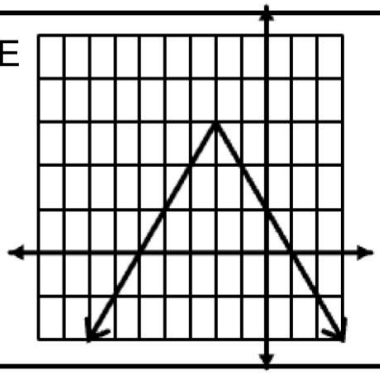
C



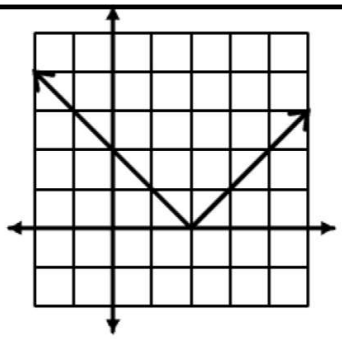
D



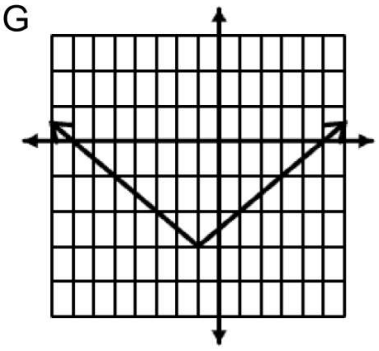
E



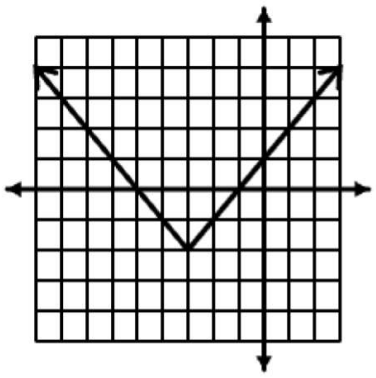
F



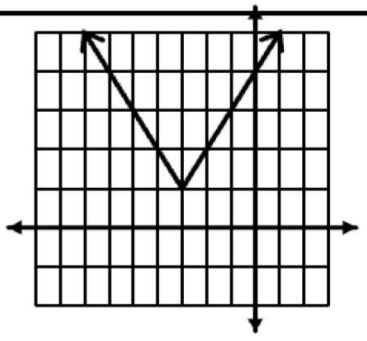
G



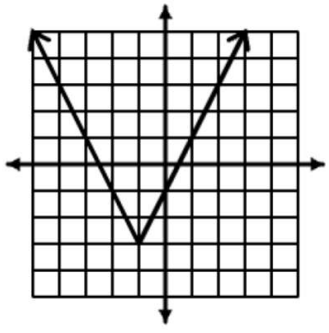
H



I

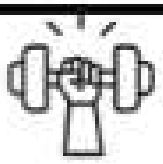


J



Clues
1. _____ I've been transformed according to the rule: g(x) = f(x + 2) **D**
2. _____ I've been transformed according to the rule: g(x) = f(x − 3) − 2 **H**
3. _____ I've been transformed according to the rule: g(x) = f(x + 3) + 1 **I**
4. _____ I've been transformed according to the rule: g(x) = −f(x + 2) + 3 **E**
5. _____ I've been transformed according to the rule: g(x) = f(x − 2 **A**
6. _____ I've been transformed according to the rule: g(x) = 2f(x + 1) − 3 **J**
7. _____ I've been transformed according to the rule: g(x) = f(x − 2) **F**
8. _____ I've been transformed according to the rule: g(x) = 0.5f(x + 1) − 3 **G**
9. _____ I've been transformed according to the rule: g(x) = f(x − 3) + 2 **C**
10. _____ I've been transformed according to the rule: g(x) = −f(x) + 2 **B**

Warm-Up



Solve.

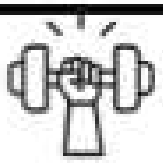
1. $\frac{2}{3}x - 4 = 8$

2. $\frac{2x-4}{6} = 3$

x = 18

x = 11

Warm-Up



Solve.

1. $\frac{2}{3}x - 4 = 8$

2. $\frac{2x-4}{6} = 3$

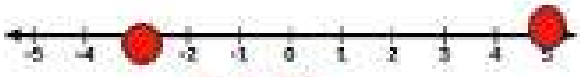
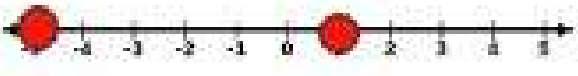
x = 18

x = 11

Solving Absolute Value Equations I

Student Handout

Example 1: Model each absolute value equation with a visual representation and then solve.

Equation	Meaning	Visual Representation
a) $\lvert x\rvert = 3$	*What number(s) are 3 units away from 0?*	 {-3, 3}
b) $\lvert x - 1\rvert = 4$	*What number(s) are 4 units away from 1?*	 {-3, 5}
c) $\lvert x + 2\rvert = 3$	*What number(s) are 3 units away from -2?*	 {-5, 1}
d) $\lvert x - 5\rvert = -4$	*What number(s) are -4 units away from 5?*	 No Solution; Distance can't be negative!

Formulating Absolute Value Equations

$$|x - c| = d$$

Center

Distance

Example 2: The graphs below represent the solutions to an absolute value equation. Write an absolute value equation that would produce the two solutions

a) 

Center = 5 Dist: 3

$|x - 5| = 3$

b) 

Center: -6 Dist: 3

$|x + 6| = 3$

c) 

Center: 2.5 Dist: 3.5

$|x - 2.5| = 3.5$

d) 

Center -2 Dist: 4

$|x + 2| = 4$

Solve Absolute Value Equations

$\|5-2x\|-11=0$		Original Equation
$\|5-2x\|=11$		Rewrite in the form $\|ax+b\|=c$
$5-2x=11$	$5-2x=-11$	Create two equations in the form
$-2x=6$	$-2x=-16$	$ax+b=c$ and $ax+b=-c$
$x=-3$	$x=8$	Solve for x

Example 3: Solve each equation below.

a) $-2|2x+3|+14=-16$

$-2|2x+3|=-30$

$|2x+3|=15$

$2x+3=15$ $\quad$ $2x+3=-15$

$2x=12$ $\quad$ $2x=-18$

$X=6$ $\quad$ $x=-9$

$\{-9, 6\}$

b) $\frac{1}{3}|-3x-3|=8$

$|-3x-3|=24$

$-3x-3=24$ $\quad$ $-3x-3=-24$

$-3x=27$ $\quad$ $-3x=-21$

$x=-9$ $\quad$ $x=7$

$\{-9, 7\}$

c) $-3\left|\frac{x+1}{2}\right|=-12$

$|(x+1)/2|=4$

$(x+1)/2=4$ $\quad$ $(x+1)/2=-4$

$X+1=8$ $\quad$ $x+1=-8$

$x=7$ $\quad$ $x=-9$

$\{-9, 7\}$

d) $3-|x+4|=5$

$-|x+4|=2$

$|x+4|=-2$

No Solution

Example 4: A machine fills oatmeal containers with 32 ounces of oatmeal. After the containers are filled, another machine weighs them. If the container's weight differs from the desired 32 ounce weight by more than 0.5 ounces, the container is rejected. Write an equation that can be used to find the heaviest and lightest acceptable weights for the oatmeal containers and then solve the equation

Center (Ideal): 32 $\quad$ Distance ("wiggle room"): 0.5

$|x-32|=0.5$

$\{31.5, 32.5\}$

Name_____ Pd_____

Exit Ticket

1. Solve $|8 + x| - 1 = 29$ Solution: (-18, 2)

2. Write an absolute value equation that would result in the two solutions shown on the number line below

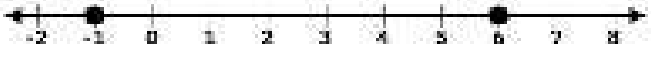
$|x - 2.5| = 3.5$

Name_____ Pd_____

Exit Ticket

1. Solve $|8 + x| - 1 = 29$ Solution: (-18, 2)

2. Write an absolute value equation that would result in the two solutions shown on the number line below



$|x - 2.5| = 3.5$

Name:_____ Period:_____

Practice

Solving Absolute Value Equations I

In 1 -4, write an absolute value equation which could be used to obtain the given solutions.

1. $|x-1| = 5$

2. $|x+3| = 5$

3. $|x-1.5| = 2.5$

4. $|x-32.5| = 12.5$

Solve each absolute value equation algebraically. Show the work that leads to your answer.

5. $2|a-17| = 10$

{12, 22}

6. $4|2-9y| = 28$

{-5/9, 1}

7. $\frac{|x-3|}{5} = 2$

{-7, 13}

8. $-\left|2-\frac{1}{3}x\right| = -9$

{-21, 33}

9. The packing material for a particular computer needs to be within 0.5 mm of the desired thickness, which is 27.5 mm. Write an absolute value equation which represents the limits of the width of the packing material.

$|x-27.5| = 0.5$

10. A factory produces widgets whose length must be within 1.5 mm of an ideal length of 47 mm. A factory supervisor wants to mark rulers with the least and greatest acceptable length for each widget for workers to use in quality control inspections. Write an absolute value equation to describe this situation and then find L, the length of the markings along the ruler.

$|L-47| = 1.5$

$L = 45.5$ $L = 48.5$

Name_____ Period_____

"Where do chickens go to work?"

Solve each equation. The answer to each problem will match a letter that will allow you to figure out the joke. Show the work that leads to your answer.

1. $|x + 6| = 15$

2. $\frac{|2x+4|}{-2} = -8$

3. $3|7x - 7| - 9 = 12$

4. $2|x + 4| = 2$

5. $|-4x - 7| + 15 = 24$

6. $\frac{3}{5}|10 - 5x| + 8 = 29$

7. $\frac{|3-6x|}{5} - 16 = -7$

8. $|4x - 8| = 36$

9. $14 - |3x + 6| = 5$

I {-6, 2}

N {-5, -3}

B {-3, 7}

G {0, 2}

M {-4, 8}

D {-6, 2}

L {-4, ½ }

T {-5, 9}

H {3, 6}

P {-7, 11}

S {-21, 9}

W {-1, 2}

G {-7, 8}

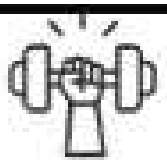
E {-5, 1}

A {-10, 6}

F {-12, 14}

E	G	G	P	L	A	N	T	S
9	3	7	8	5	2	4	6	1

Warm-Up



Sarah worked the problem below incorrectly. Describe her error and then solve the problem correctly.

$2|x-2| = 12$

$|2x-4| = 12$

$2x-4 = 12 \quad 2x-4 = -12$

$2x = 16 \quad 2x = -8$

$x = 8 \quad x = -4$

$\{-4, 8\}$

Describe the error: **Sarah made a mistake in step one by distributing the 2. She should have divided both sides by 2**

Solve $2|x-2| = 12$ correctly:

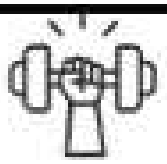
$$|x-2| = 6$$

$$x-2 = 6 \quad x-2 = -6$$

$$x = 8 \quad x = -4$$

$$\{-4, 8\}$$

Warm-Up



Sarah worked the problem below incorrectly. Describe her error and then solve the problem correctly.

$2|x-2| = 12$

$|2x-4| = 12$

$2x-4 = 12 \quad 2x-4 = -12$

$2x = 16 \quad 2x = -8$

$x = 8 \quad x = -4$

$\{-4, 8\}$

Describe the error: **Sarah made a mistake in step one by distributing the 2. She should have divided both sides by 2**

Solve $2|x-2| = 12$ correctly:

$$|x-2| = 6$$

$$x-2 = 6 \quad x-2 = -6$$

$$x = 8 \quad x = -4$$

$$\{-4, 8\}$$

Solving Absolute Value Equations II

Identifying Special Solutions

When you solve an absolute value equation, it is possible for a solution to be ***extraneous***. An **extraneous solution** is an apparent solution that must be rejected because it does not satisfy the original equation.

Example 1: Solve each of the following and check for extraneous solutions

a) $|2x + 12| = 4x$ **{6}**

b) $|2x - 3| = 4x - 1$ **{2/3}**

c) $-3\left|\frac{x+1}{2}\right| + 5 = -7$ **{-9, 7}**

d) $|2x + 5| = 3x + 4$ **{1}**

Example 2: The number of shoppers in a store is modeled by the equation $s(t) = -0.5|t - 288| + 144$ where t is the time (in minutes) since the store opened at 10:00 A.M.

a) How many shoppers are in the store at 10:30 A.M?

15

b) At what times are there 100 shoppers in the store?

200 minutes and 376 minutes

3 hours 20 minutes and 6 hours and 16 minutes

1:20 P.M. 4:16 P.M

Name_____ Pd_____

1.Solve the equation $|4x + 1| = 2x + 9$ and check for extraneous solutions.

{-5/3,4}

2. Sam is working a part-time job painting houses. He averages 7.5 houses per week. The difference between his average number of houses per week and the actual number of houses he paints per day is at most 2.5. Write and solve an absolute value equation to determine the minimum and maximum number of houses he paints per week.

$|x - 7.5| = 2.5$

{5, 10}

Name_____ Pd_____

Exit
Ticket

1.Solve the equation $|4x + 1| = 2x + 9$ and check for extraneous solutions.

{-5/3,4}

2. Sam is working a part-time job painting houses. He averages 7.5 houses per week. The difference between his average number of houses per week and the actual number of houses he paints per day is at most 2.5. Write and solve an absolute value equation to determine the minimum and maximum number of houses he paints per week.

$|x - 7.5| = 2.5$

{5, 10}

Name_____ Period_____

Practice

Solving Absolute Value Equations II

In 1 – 4, solve each equation. Check for extraneous solutions.

1. $|2x+5| = 3x+4$

{4}

2. $|3x+4| = x+2$

{-3/2, -1}

3. $3\left|1-\frac{1}{4}x\right| = 9$

{-8, 16}

4. $6|5x-4|-4 = 32$

{-1/2, 2}

5. Over the course of 2 days last winter, the temperature in a city can be modeled by the function below, where h is the time in hours.

$$t(h) = 2\left|\frac{h}{2} - 12\right|$$

a) Find the temperature at h = 8 hours **16 degrees**

b) Determine the times when the temperature is equal to 20 degrees. **h = 4 and h = 44**

6. Two different points on a number line are both 3 units from the point with coordinate -4. Create a sketch for this situation and then choose the equation below which best models this situation.

a) $|x+4| = 3$ b) $|x-4| = 3$ c) $|x+3| = 4$ d) $|x-3| = 4$

A

7. What is the value of k that satisfies both equations below?

$$10-|k| = 3$$
$$|k-5| = 8$$

10 – k = 3 **10 – k = -3**
k = 7 **k = 13**

k – 5 = 8 **k – 5 = -8**
k = 13 **k = -3**

k = 13

8. If $2|x-9| = 20$,, what is the sum of the solutions to the given equation?

x - 9 = 10 **x – 9 = -10**
x = 19 **x = -1**
19 + -1 = 18

Warm-Up

Solve each inequality and sketch the graph of the solution set

a) $5 - 2x > 13$

$x < -4$

b) $\frac{1}{2}x + 4 \geq 6$

$x \geq 4$

Warm-Up

Solve each inequality and sketch the graph of the solution set

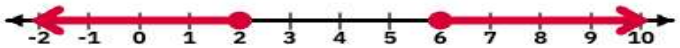
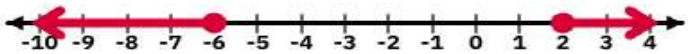
a) $5 - 2x > 13$

b) $\frac{1}{2}x + 4 \geq 6$

Notes

Solving Absolute Value Inequalities

Example 1: Model each absolute value inequality on the number line than identify the solution set.

Inequality	Meaning	Model and Solve
a) $\lvert x\rvert \leq 3$	"What numbers have distance from 0 that is less than or equal to 3 units?	 $x \geq$ -3 and $x \leq$ 3 Solution: -3 ≤ x ≤ 3
b) $\lvert x-1\rvert \leq 4$	"What numbers have distance from 1 that is less than or equal to 4 units?	 $x \geq$ -4 and $x \leq$ 5 Solution: -4 ≤ x ≤ 5
c) $\lvert x-4\rvert \geq 2$	"What numbers have distance from 4 that is greater than or equal to 2 units?	 Solution: x ≤ 2 or x ≥ 6
d) $\lvert x+2\rvert \geq 4$	"What numbers have distance from -2 that is greater than or equal to 4 units?	 Solution: x ≤ -6 or x ≥ 2

Example 2: The normal human body temperature is 98.6°F. A temperature, x, that differs from normal by 2° or more is considered unhealthy. Write an absolute value inequality to describe an unhealthy body temperature.

$|x - 98.6| \geq 2$

Example 3: The packing material for a particular computer needs to be within 0.5 mm of the desired thickness, which is 27.5 mm. Write an absolute value inequality to describe the range of acceptable values for the thickness of the packing material.

$|x - 27.5| \leq 0.5$

When solving absolute value inequalities, just remember "GOLA"!
Greater than: "Or"
Less than: "And"

Problem	Rewrite as:	Example
$\|ax + b\| > c$	$ax + b > c$ OR $ax + b < -c$	$\|2x + 3\| > 12$ $2x + 3 > 12$ OR $2x + 3 < -12$
$\|ax + b\| < c$	$-c < ax + b < c$	$\|2x + 3\| < 12$ $-12 < 2x + 3 < 12$
Special Cases		
$\|ax + b\| < -c$	No Solution	$\|2x + 3\| < -12$ Distance can not be less than a negative
$\|ax + b\| > -c$	All Real Numbers	$\|2x + 3\| < -12$ Distance will ALWAYS be greater than a negative!

Example 3: Solve each absolute value inequality and graph the solution set.

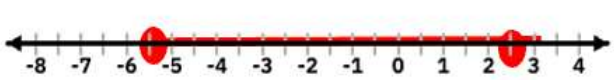
a) $-2|2x + 3| \geq -16$

$|2x + 3| \leq 8$

$-8 < 2x + 3 < 8$

$-11 < 2x < 5$

$-11/2 < x < 5/2$

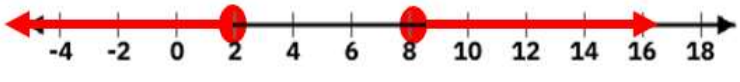


b) $6 + 2|x - 5| > 12$

$|x - 5| > 3$

$x - 5 > 3$ or $x - 5 < -3$

$x > 8$ or $x < 2$

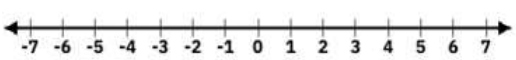


c) $8|x + 4| + 10 < 2$

$|x + 4| < -1$

Distance can never be less than -1!

No Solution

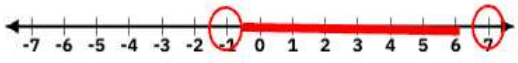


d) $5 - |x - 3| - 3 > -2$

$|x - 3| < 4$

$-4 < x - 3 < 4$

$-1 < x < 7$



Name_____ Pd_____

Solve each inequality and graph the solution set

a) $3|x-4|+2>5$

$|x-4|>1$

$x-4 \geq 1$ or $x-4 \leq -1$

$x \geq 5$ or $x \leq 3$

b) $4-|x+2|>2$

$|x+2|<2$

$-2<x+2<2$

$-4<x<0$

Name_____ Period_____

Practice Solving Absolute Value Inequalities

Solve each inequality and graph the solution.

1. $|4 - x| + 15 > 21$

$x < -2$ or $x > 10$

2. $|x + 4| - 10 \leq -2$

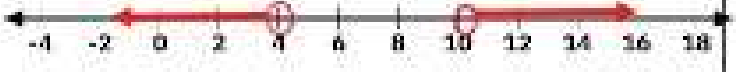
$-12 \leq x \leq 4$

3. $-3|x + 2| + 6 > -6$

$-6 < x < 2$

4. $3|x - 7| \geq 9$

$x \geq 10$ or $x \leq 4$



5. $|2x + 3| < 5$

$-4 < x < 1$

6. $5 - 6|x + 7| \leq 17$

No Solution

7. Your car averages 32 miles per gallon in the city. The actual mileage varies from the average by at most 5 miles per gallon.

a) Write an absolute value inequality that shows the range for the mileage your car gets.

$|x - 32| \leq 5$

b) Solve the inequality to determine range of miles per gallon your car will get in the city.

$27 \leq x \leq 37$

8. Members of a track team can run 400 m in an average of 58.2 seconds. The fastest and slowest times vary from the average at most 6.4 seconds.

a) Write an absolute value inequality that models the range of times the team can run 400 m.

$|x - 58.2| \leq 6.4$

b) Solve the inequality to find the range of times.

$51.8 \leq x \leq 64.6$

Name_____-_____Period_____

Test Review

Absolute Value

1. Some values of an absolute value function are shown in the table below

x	0	1	2	3	4	5	6
f(x)	2	3	4	5	4	3	2

a) What is the axis of symmetry? x = 3

b) What is the vertex? (3, 5)

c) What is the y-intercept? (0, 2)

d) What is the maximum value of the function? 5

2. An absolute value function is shown below. Find each of the following.

a) What is the axis of symmetry? x = -2

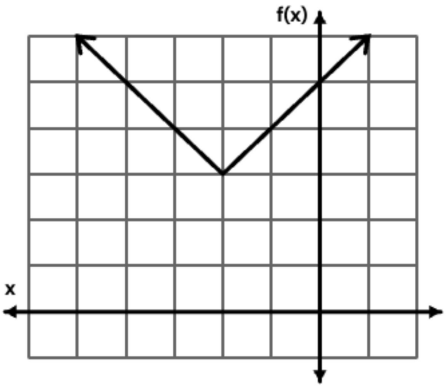
b) What is the vertex? (-2, 3)

c) What is f(-1)? 4

d) What is the domain and range?

Domain: R Range; y ≥ 3

e) What is the minimum value of the function? 3



3. An absolute value function is shown below. Find each of the following

$$f(x) = -2|x - 5| + 4$$

a) What is the axis of symmetry? x = 5

b) What is the vertex? (5, 4)

c) What is f(-1)? -8

d) Domain: R Range; y ≤ 4

e) What is the maximum value of the function? 4

Describe the transformations necessary to transform f(x) = |x| to obtain g(x)

4. $g(x) = 2|x - 3| + 2$

Vertical Stretch bfo 2

Right 3

Up 2

5. $g(x) = -\frac{1}{2}|x + 3| - 5$

Reflect across x-axis

Vert. Compression bfo ½

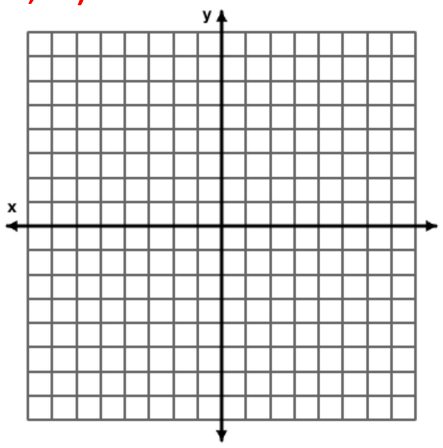
Left 3

Down 5

State the vertex, domain, range, and then graph each of the following.

6. $f(x) = -2|x + 1| + 2$

Vertex: (-1, 2)

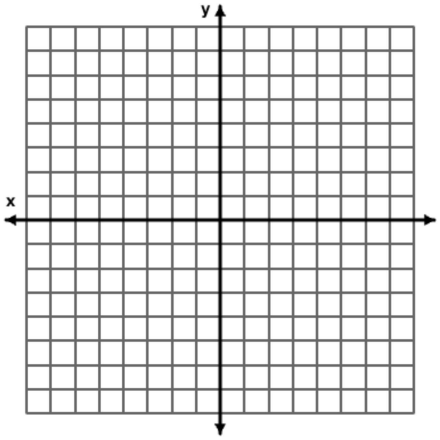


Domain: R

Range: y ≤ 2

7. $f(x) = \frac{1}{2}|x - 2| - 4$

Vertex: (2, -4)

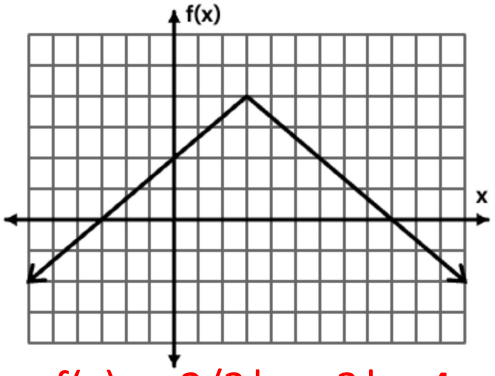


Domain: R

Range: y ≥ -4

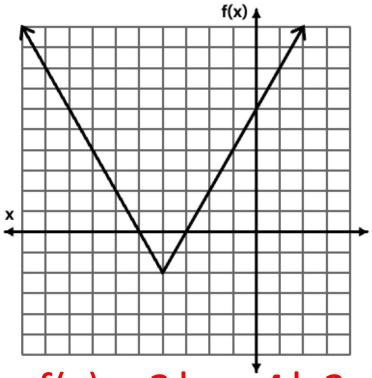
Write an equation to describe each graph

8.



Equation: f(x) = -2/3|x – 3| + 4

9.



Equation: f(x) = 2|x + 4|-2

Solve each of the following

10. $-2|7-3w|-6=-14$

{1, 11/3}

11. $4|2y-7|+5=9$

{3, 4}

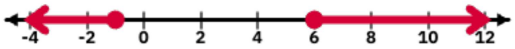
Solve and graph the solution set for each of the following.

12. $\left|\frac{1}{2}x-2\right| \leq 3$

Solution: -2 < x < 10

13. $|2x-5| \geq 7$

Solution: x ≤ -1 or x ≥ 6



14. The absolute value equation below has two solutions which can be written in the form {a, b}. \ What is the value of a + b?

$5-3|2+2x|=-7$

-3|2 + 2x| = -12
|2 + 2x| = 4
2 + 2x = 4 2 + 2x = -4
2x = 2 2x = -6
x = 1 x = -3

{-3, 1}
a + b = -3 + 1

= -2

15. Your car averages 28 miles per gallon in the city. The actual mileage varies from the average by at most 5 miles per gallon. Write an absolute value inequality that shows the range for the mileage your car gets.

Inequality: |x − 28| ≤ 5

16. The average test score on the last algebra II test was a 78. If the tests scores were all within 22 points of the average score, write an absolute value inequality that describes this situation and then solve.

Inequality |x − 78| ≤ 22

Solution: 56 ≤ x ≤ 100

Name_____ Period_____

Unit Test Form A

Absolute Value

1. The function $f(x) = |x + 3| + 5$ is symmetrical about which line?

A $x = 3$
B $y = 3$
C $x = -3$
D $y = -3$

C

2. The function $f(x) = |x - 4| + 2$ has a vertex of:

A $(4, 2)$
B $(2, 4)$
C $(-4, 2)$
D $(2, -4)$

A

3. What is the y-intercept for the function below?

$$g(x) = |x - 3| + 2$$

A (0, 2) C (2, 0)

B (0, 5) D (5, 0)

B

4. What is the maximum value of the function below?

$$f(x) = -|x - 10| + 6$$

A 6 C 10

B 12 D 8

A

5. Evaluate the function below for f(-2)

$$f(x) = |x - 3| + 2$$

A -3 C 7

B 3 D Not Here

C

6. Which of the following is not true about the function $g(x) = -|x - 2| + 3$?

A The vertex is (2, 3)
B The function is symmetric about the line x = 2
C. The function has a y-intercept of (0, 1)
D. The function has a maximum value of 2

D

7. When compared to the parent function, $f(x) = |x|$, which equation has been vertically stretched and shifted left?

A $g(x) = \frac{3}{4}|x + 2|$ C $g(x) = \frac{3}{4}|x - 4|$

B $g(x) = \frac{5}{2}|x + 4|$ D $g(x) = \frac{5}{2}|x - 4|$

B

8. The vertex of absolute value function, $f(x)$ is (-1, 6). What is the vertex of g(x) if it has the transformation rule $g(x) = f(x - 3) + 5$?

A (-4, 11) C (2, 11)

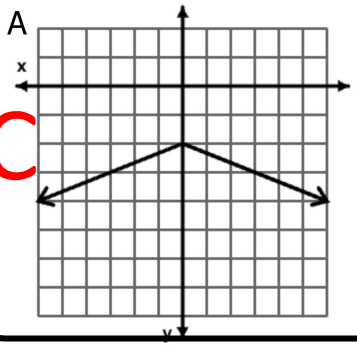
B (2, 1) D (-4, 1)

C

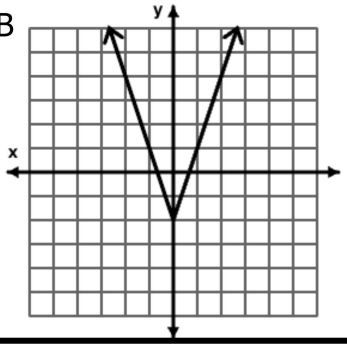
9. Avery transformed the function f(x) = |x| according to the rule g(x) = -3f(x) – 2. Which of the following graphs correctly shows this transformation?

C

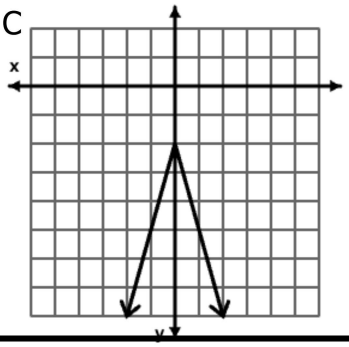
A



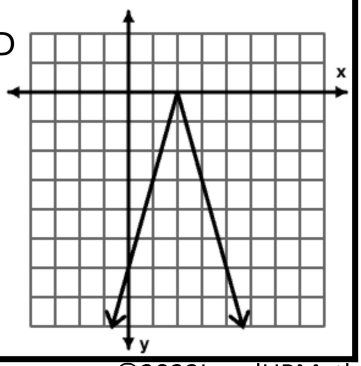
B



C



D



10. What is the solution set of the equation below?

$$2|x - 3| = 12$$

{-3, 9}

11. What is the solution set of the equation below?

$$3|5 - x| + 2 = 29$$

{-4, 14}

12. The absolute value equation below has two solutions which can be written in the form {a, b}. What is the value of a + b?

$$3\left|1 - \frac{1}{4}x\right| = 12$$

{-12, 20}

13. What is the value of *k* that satisfies both equations below?

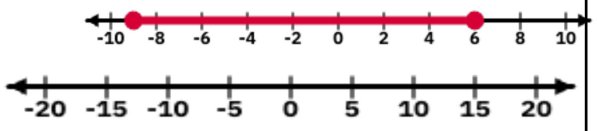
$$10 - k| = 3$$
$$|k - 5| = 8$$

k = 13

14. Solve and graph $|4x + 6| \leq 30$

$-9 \leq x \leq 6$

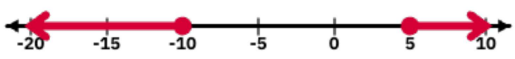
Solution:_____



15. Solve and graph $|2x + 5| \geq 15$

$x \leq -10$ or $x \geq 5$

Solution:_____

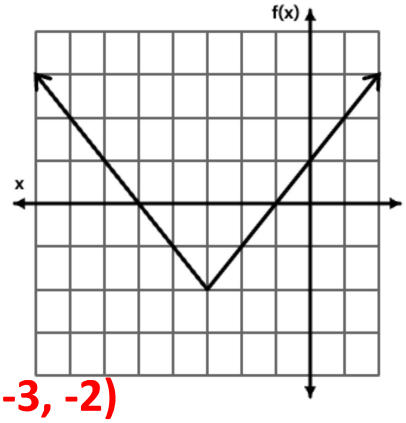


16. The bus routes in a city run on average every 15 minutes. The route times can vary by three minutes. Which absolute value equation can be used to find the maximum and minimum wait times at a bus stop?

A

A $|x - 15| = 3$ B $|x + 15| = 3$ C $|x + 3| = 15$ D $|x - 3| = 15$

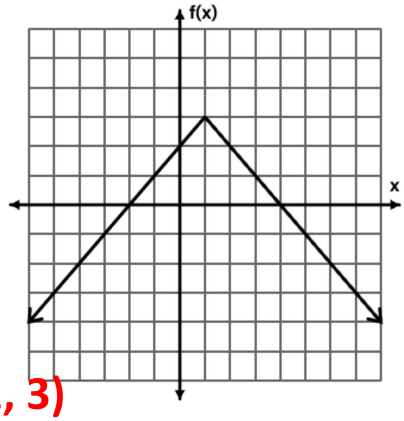
17. Identify the vertex and then write an absolute value function to describe f(x) shown in the graph below.



Vertex: **(-3, -2)**

Equation: **f(x) = |x + 3| - 2**

18. Identify the vertex and then write an absolute value function to describe f(x) shown in the graph below.

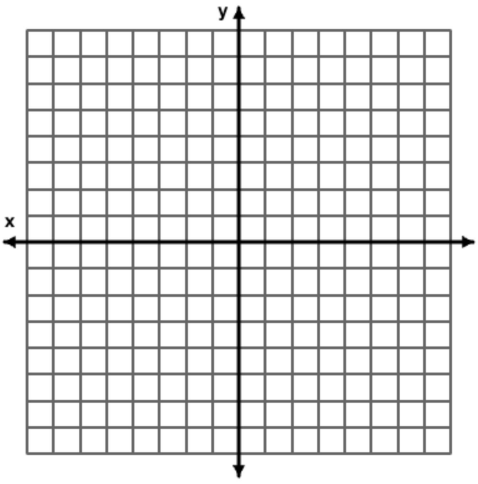


Vertex: **(1, 3)**

Equation: **f(x) = -|x – 1| + 3**

19. Give the vertex and then graph the function. State the domain and range.

$$g(x) = 2|x - 2| - 3$$



Vertex: **(2, -3)**

Domain: **R**

Range: **y ≥ -3**

Bonus: Let a and b both be negative numbers such that $|2a - 3| = 5$ and $|3 - 4b| = 11$. What is $|b - a|$? Show the work that leads to your answer. No Work, No Credit, No Kidding!

$|b - a| = |-2 - (-1)| = |-1| = 1$

Name_____ - _____ Period_____

Unit Test Form B

Absolute Value

1. The function $f(x) = |x + 3| + 5$ is symmetrical about which line?

A $y = -3$
B $x = 3$
C $y = 3$
D $x = -3$

D

2. The function $f(x) = |x - 4| + 2$ has a vertex of:

A $(2, -4)$
B $(4, 2)$
C $(2, 4)$
D $(-4, 2)$

B

3. What is the y-intercept for the function below?

$$g(x) = |x - 3| + 2$$

A (2, 0)
C (0, 2)
B (5, 0)
D (0, 5)

D

4. What is the maximum value of the function below?

$$f(x) = -|x - 10| + 6$$

A 8
C 10
B 12
D 6

D

5. Evaluate the function below for f(-3)

$$f(x) = |x - 3| + 2$$

B

A -4
C 2
B 8
D Not Here

6. Which of the following is not true about the function $g(x) = -|x - 2| + 3$?

A

A. The function has a maximum value of 2
B The vertex is (2, 3)
C The function is symmetric about the line x = 2
D The function has a y-intercept of (0, 1)

7. When compared to the parent function, $f(x) = |x|$, which equation has been vertically compressed and shifted left?

A $g(x) = \frac{3}{4}|x + 2|$
C $g(x) = \frac{3}{4}|x - 4|$
B $g(x) = \frac{5}{2}|x + 4|$
D $g(x) = \frac{5}{2}|x - 4|$

A

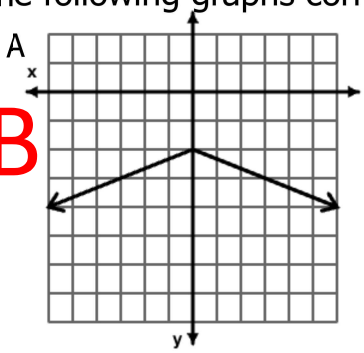
8. The vertex of absolute value function, $f(x)$ is (-1, 6). What is the vertex of g(x) if it has the transformation rule $g(x) = f(x - 3) + 5$?

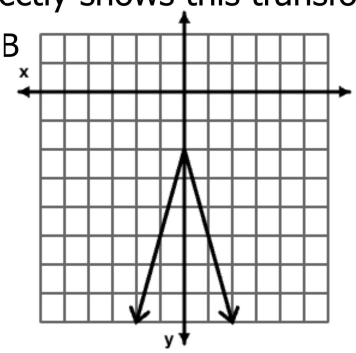
A (-4, 1)
C (2, 11)
B (2, 1)
D (-4, 11)

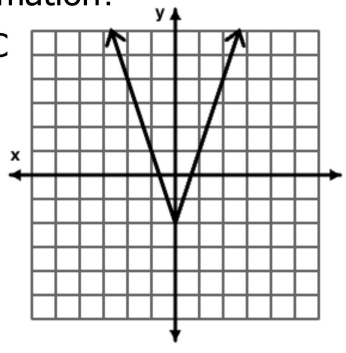
C

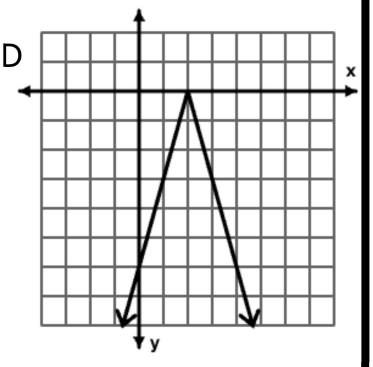
9. Avery transformed the function f(x) = |x| according to the rule g(x) = -3f(x) – 2. Which of the following graphs correctly shows this transformation?

B

A 

B 

C 

D 

10. What is the solution set of the equation below?

$$2|x - 3| = 14$$

{-4, 10}

11. What is the solution set of the equation below?

$$3|4 - x| + 2 = 17$$

{-1, 9}

12. The absolute value equation below has two solutions which can be written in the form {a, b}. What is the value of a + b?

$$3\left|1 - \frac{1}{4}x\right| = 15$$

{-16, 24}

13. What is the value of *k* that satisfies both equations below?

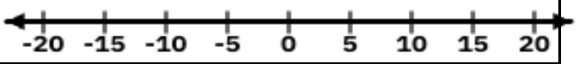
$$10 - k| = 3$$
$$|k - 5| = 8$$

k = 13

14. Solve and graph $|4x + 6| \geq 30$

$-9 \leq x \leq 6$

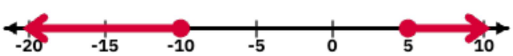
Solution:_____



15. Solve and graph $|2x + 5| \leq 15$

$x \leq -10$ or $x \geq 5$

Solution:_____

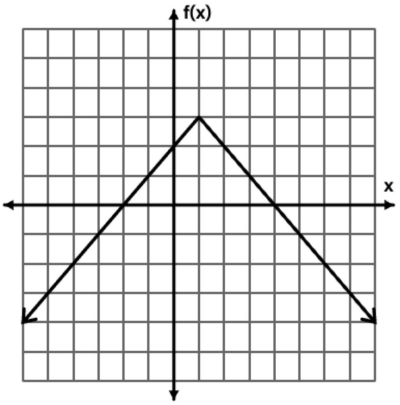


16. The bus routes in a city run on average every 15 minutes. The route times can vary by three minutes. Which absolute value equation can be used to find the maximum and minimum wait times at a bus stop?

D

A $|x - 3| = 15$ B $|x + 15| = 3$ C $|x + 3| = 15$ D $|x - 15| = 3$

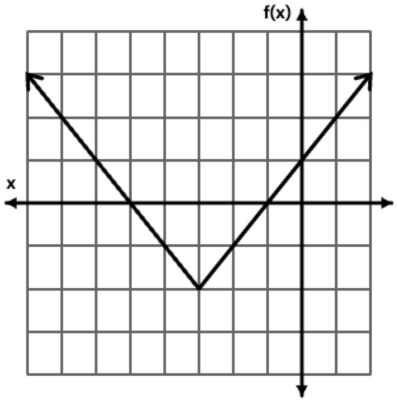
17. Identify the vertex and then write an absolute value function to describe f(x) shown in the graph below.



Vertex: **(1, 3)**

Equation: **f(x) = -|x - 1| + 3**

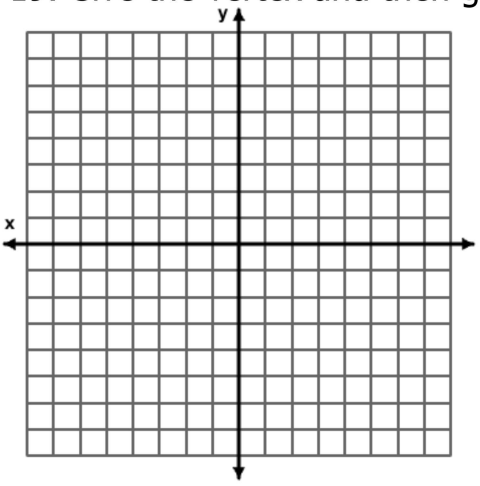
18. Identify the vertex and then write an absolute value function to describe f(x) shown in the graph below.



Vertex: **(-3, -2)**

Equation: **f(x) = |x + 3| -2**

19. Give the vertex and then graph the function. State the domain and range.



$g(x) = 2|x - 2| - 3$

Vertex: **(2, -3)**

Domain: **R**

Range: **y > -3**

Bonus: Let a and b both be negative numbers such that $|2a - 3| = 5$ and $|3 - 4b| = 11$. What is $|b - a|$? Show the work that leads to your answer. No Work, No Credit, No Kidding!

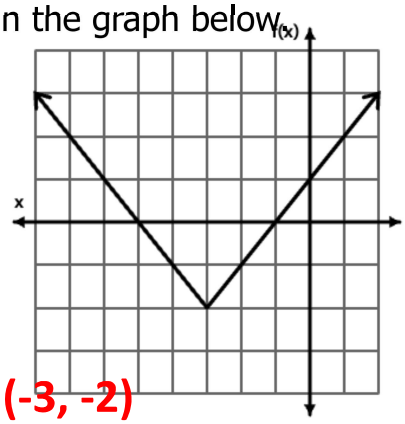
$|b - a| = |-2 - (-1)| = |-1| = 1$

16. The bus routes in a city run on average every 15 minutes. The route times can vary by three minutes. Which absolute value equation can be used to find the maximum and minimum wait times at a bus stop?

A

A $|x - 15| = 3$ B $|x + 15| = 3$ C $|x + 3| = 15$ D $|x - 3| = 15$

17. Identify the vertex and then write an absolute value function to describe f(x) shown in the graph below.

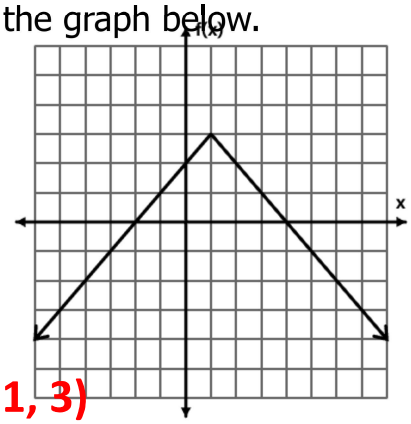


(-3, -2)

Vertex: **f(x) = |x + 3| - 2**

Equation: _____

18. Identify the vertex and then write an absolute value function to describe f(x) shown in the graph below.

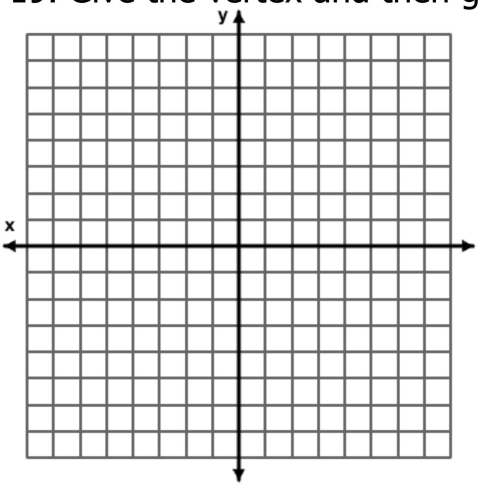


(1, 3)

Vertex: **f(x) = -|x – 1| + 3**

Equation: _____

19. Give the vertex and then graph the function. State the domain and range.



$g(x) = 2|x - 2| - 3$

(2, -3)

Vertex: _____

Domain: **R**

Range: **y ≥ -3**

Bonus: Let a and b both be negative numbers such that $|2a - 3| = 5$ and $|3 - 4b| = 11$. What is $|b - a|$? Show the work that leads to your answer. No Work, No Credit, No Kidding!

$|b - a| = |-2 - (-1)| = |-1| = 1$