

Name:

Date:

Topic:

Class:

Main Ideas/Questions	Notes/Examples			
<b>POLYGON</b>	A polygon is a <u>closed</u> figure formed by three or more <u>line segments</u> , called <u>sides</u> .			
<b>Sum of the INTERIOR ANGLE Measures</b>	The sum of the measures of the interior angles in any polygon can be determined by the number of triangles that can be drawn within the polygon. Complete the table below and look for a pattern to find the sum of the degrees in any polygon.			
Polygon	Picture	# of Sides	# of Triangles	Sum of Interior∠'s
Triangle		3	1	180°
Quadrilateral		4	2	360°
Pentagon		5	3	540°
Hexagon		6	4	720°
Heptagon	X	7	5	900°
Octagon	X	8	6	1080°
Nonagon	X	9	7	1260°
Decagon	X	10	8	1440°
<b>INTERIOR Angle Sum FORMULA</b>	If $n$ represents the number of sides of a polygon, then the sum of the interior angle, $S$ , can be found using the formula: $S = (n-2) \cdot 180$			
<b>Find the sum of the measures of the interior angles in each polygon.</b>				
1. 15-gon $S = (15-2) \cdot 180 = 2340^\circ$	2. 21-gon $S = (21-2) \cdot 180 = 3420^\circ$			
3. 48-gon $S = (48-2) \cdot 180 = 8280^\circ$	4. 36-gon $S = (36-2) \cdot 180 = 6120^\circ$			

## REGULAR POLYGON

A polygon in which all side measures are congruent, therefore all angle measures are congruent.

The measure of a single interior angle in a regular polygon can be found by dividing the sum of the interior angle measures,  $S$ , by the number of sides,  $n$ .

Find the measure of each interior angle in the following polygons.

5. regular pentagon

$$S = \frac{(5-2) \cdot 180}{5} = \frac{540}{5} = 108^\circ$$

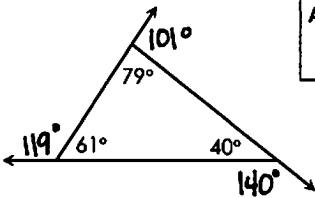
6. regular 18-gon

$$S = \frac{(18-2) \cdot 180}{18} = \frac{2880}{18} = 160^\circ$$

## Sum of the EXTERIOR ANGLE Measures

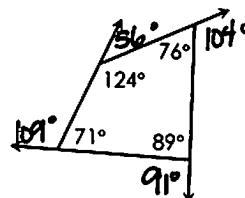
Exterior angles are supplementary to their adjacent interior angle. Find the measure of each exterior angle on the polygons below, then give the sum of all exterior angle measures.

Triangle:



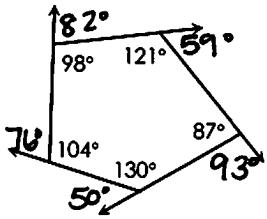
Sum of Exterior Angles Measures:  
360°

Quadrilateral:



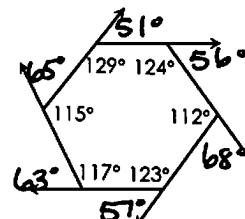
Sum of Exterior Angles Measures:  
360°

Pentagon:



Sum of Exterior Angles Measures:  
360°

Hexagon:



Sum of Exterior Angles Measures:  
360°

What can you conclude about the sum of the exterior angles measures of a polygon?

The sum of the exterior angles of any polygon is 360°.

## MORE EXAMPLES

7. What is the measure of each exterior angle of a regular hexagon?

$$\frac{360}{6} = 60^\circ$$

8. What is the measure of each exterior angle of a regular 24-gon?

$$\frac{360}{24} = 15^\circ$$

9. If the exterior angle of a regular polygon measures 12°, how many sides does the polygon have?

$$\frac{360}{X} = 12$$

$$12X = 360  
X = 30$$

10. If the exterior angle of a regular polygon measures 40°, how many sides does the polygon have?

$$\frac{360}{X} = 40$$

$$40X = 360  
X = 9$$

more practice with

# ● ● INTERIOR & EXTERIOR ANGLE MEASURES ● ●

## INTERIOR & EXTERIOR OF ANY POLYGON REFERENCE:

Sum of the INTERIOR Angle Measures:

$$S = (n-2) \cdot 180$$

Sum of the EXTERIOR Angle Measure:

$$360^\circ$$

## INTERIOR & EXTERIOR ANGLES OF REGULAR POLYGONS REFERENCE:

Interior Angle Measure  
of a Regular Polygon:

$$\frac{S}{n} = \frac{(n-2) \cdot 180}{n}$$

Exterior Angle Measure  
of a Regular Polygon:

$$\frac{360}{n}$$

The Number of Sides  
of a Regular Polygon:

$$\frac{360}{\text{ext } \angle}$$

## PRACTICE QUESTIONS

1. What is the sum of the measures of the interior angles of a pentagon?

$$S = (5-2) \cdot 180 = 540^\circ$$

2. What is the sum of the measures of the interior angles of a 27-gon?

$$S = (27-2) \cdot 180 = 4500^\circ$$

3. What is the measure of each interior angle of a regular octagon?

$$S = (8-2) \cdot 180 = 1080$$
$$1080/8 = 135^\circ$$

4. What is the measure of each interior angle of a regular 20-gon?

$$S = (20-2) \cdot 180 = 3240$$
$$3240/20 = 162^\circ$$

5. Five angles of a hexagon measure  $119^\circ$ ,  $129^\circ$ ,  $104^\circ$ ,  $139^\circ$ , and  $95^\circ$ . What is the measure of the sixth angle?

$$S = (6-2) \cdot 180$$
$$= 720^\circ$$

$$720 - 586$$
$$= 134^\circ$$

6. The sum of the interior angles of a polygon is  $1620^\circ$ . How many sides does the polygon have?

$$(n-2) \cdot 180 = 1620$$

$$n-2 = 9$$

$$n = 11 \text{ sides}$$

7. The sum of the interior angles of a polygon is  $3960^\circ$ . How many sides does the polygon have?

$$(n-2) \cdot 180 = 3960$$

$$n-2 = 22$$

$$n = 24 \text{ sides}$$

8. What is the sum of the measures of the exterior angles of a nonagon?

$$360^\circ$$

9. What is the measure of each exterior angle of a regular 20-gon?

$$\frac{360}{20} = 18^\circ$$

10. If the exterior angle of a regular polygon measures  $9^\circ$ , how many sides does the polygon have?

$$\frac{360}{n} = 9$$

$$9n = 360$$

$$n = \boxed{40 \text{ sides}}$$

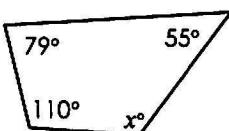
11. If the interior angle of a regular polygon measures  $108^\circ$ , how many sides does the polygon have?  $\text{ext} = 72^\circ$

$$\frac{360}{n} = 72$$

$$360 = 72n$$

$$n = \boxed{5 \text{ sides}}$$

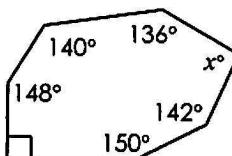
12. Find the value of  $x$ .



$$360 - 244$$

$$= \boxed{116^\circ}$$

13. Find the value of  $x$ .



$$900 - 806$$

$$= \boxed{94^\circ}$$

14. Solve for  $x$ .

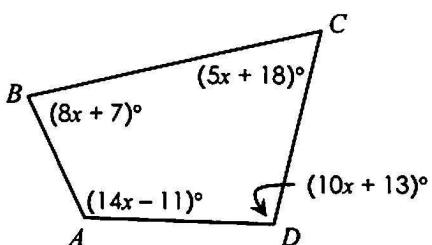
$$110 + 8x - 1 + 5x + 36 + 116 + 7x + 19 + 6x - 2 = 720$$

$$26x + 278 = 720$$

$$26x = 442$$

$$\boxed{x = 17}$$

15. Find  $m\angle B$ .



$$8x + 7 + 5x + 18 + 10x + 13 + 14x - 11 = 360$$

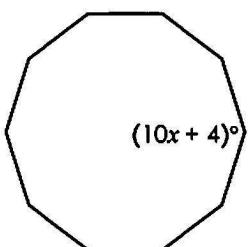
$$37x + 27 = 360$$

$$37x = 333$$

$$x = 9$$

$$m\angle B: 8(9) + 7 = \boxed{79^\circ}$$

16. If the figure below is a regular polygon, find the value of  $x$ .



$$10(10x + 4) = 1440$$

$$100x + 40 = 1440$$

$$100x = 1400$$

$$\boxed{x = 14}$$

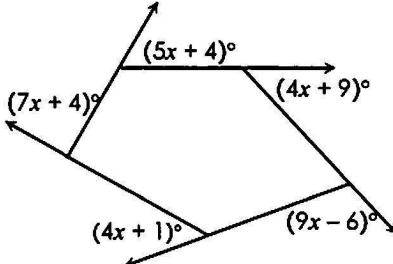
17. Find the value of  $x$ .

$$5x + 4 + 4x + 9 + 9x - 6 + 4x + 1 + 7x + 4 = 360$$

$$29x + 12 = 360$$

$$29x = 348$$

$$\boxed{x = 12}$$



Name: \_\_\_\_\_

**Unit 8: Polygons & Quadrilaterals**

Date: \_\_\_\_\_ Per: \_\_\_\_\_

**Homework 1: Angles of Polygons****\*\* This is a 2-page document! \*\***

1. What is the sum of the measures of the interior angles of an octagon?  $1080^\circ$
2. What is the sum of the measures of the interior angles of a 25-gon?  $4140^\circ$
3. What is the measure of each interior angle of a regular hexagon?  $120^\circ$
4. What is the measure of each interior angle of a regular 16-gon?  $157.5^\circ$
5. What is the sum of the measures of the exterior angles of a decagon?  $360^\circ$
6. What is the measure of each exterior angle of a regular 30-gon?  $12^\circ$
7. An exterior angle of a regular polygon measures  $22.5^\circ$ . How many sides does it have?  $16$
8. An interior angle of a regular polygon measures  $170^\circ$ . How many sides does it have?  $36$   
 $\hookrightarrow \text{ext} = 10^\circ$
9. If the sum of the measures of the interior angles of a polygon is  $1980^\circ$ , how many sides does the polygon have?  

$$(n-2) \cdot 180 = 1980$$
  

$$n-2 = 11$$
  

$$\boxed{n=13}$$
10. If the sum of the measures of the interior angles of a polygon is  $5400^\circ$ , how many sides does the polygon have?  

$$(n-2) \cdot 180 = 5400$$
  

$$n-2 = 30$$
  

$$\boxed{n=32}$$
11. The measure of the seven angles in a nonagon measure  $138^\circ$ ,  $154^\circ$ ,  $145^\circ$ ,  $132^\circ$ ,  $128^\circ$ ,  $147^\circ$ , and  $130^\circ$ . If the two remaining angles are equal in measure, what is the measure of each angle?  

$$S = (9-2) \cdot 180$$
  

$$= 1260$$
  

$$x + x + 974 = 1260$$
  

$$2x + 974 = 1260$$
  

$$2x = 286$$
  

$$\boxed{x = 143^\circ}$$
12. Find the value of  $x$ .  $S = (5-2) \cdot 180$   

$$= 540$$
  

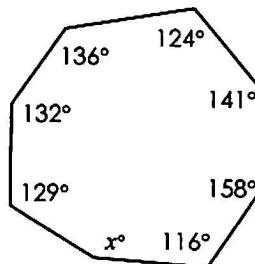
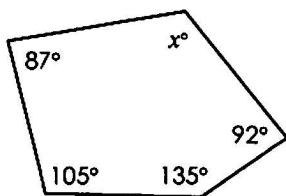
$$540 - 419$$
  

$$= \boxed{121^\circ}$$
13. Find the value of  $x$ .  $S = (8-2) \cdot 180$   

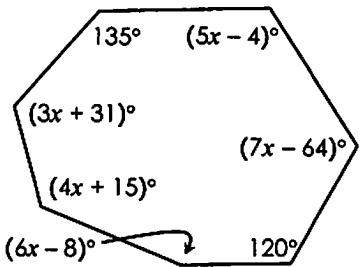
$$= 1080$$
  

$$1080 - 936$$
  

$$= \boxed{144^\circ}$$



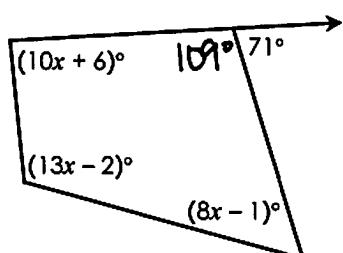
14. Find the value of  $x$ .



$$S = (7-2) \cdot 180 \\ = 900$$

$$25x + 225 = 900 \\ 25x = 675 \\ x = 27$$

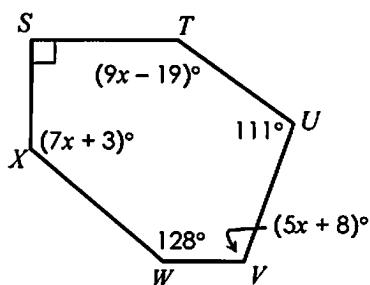
15. Find the value of  $x$ .



$$S = (4-2) \cdot 180 \\ = 360$$

$$31x + 112 = 360 \\ 31x = 248 \\ x = 8$$

16. Find  $m\angle V$ .

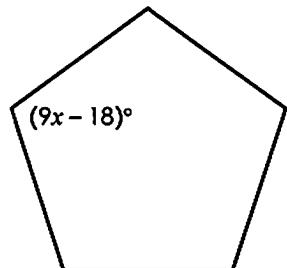


$$S = (6-2) \cdot 180 \\ = 720$$

$$21x + 321 = 720 \\ 21x = 399 \\ x = 19$$

$$m\angle V: 5(19) + 8 = 103^\circ$$

17. If the figure below is a regular polygon, find the value of  $x$ .



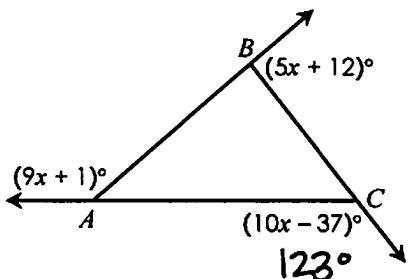
$$S = (5-2) \cdot 180 \\ = 540$$

$$9x - 18 = 108 \\ 9x = 126$$

$$\frac{540}{5} = 108$$

$$x = 14$$

18. Find  $m\angle BCA$ .



$$24x - 24 = 360 \\ 24x = 384 \\ x = 16$$

$$m\angle BCA = 180 - 123 \\ = 57^\circ$$

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## Main Ideas/Questions

## Properties of PARALLELOGRAMS

## Notes/Examples

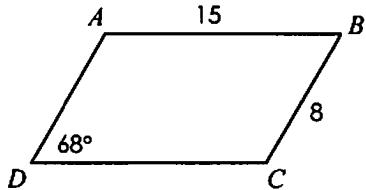
**Definition of a Parallelogram:**

A quadrilateral in which both pairs of opposite sides are parallel.  
Other important properties of parallelograms:

- ① Opposite sides are congruent.
- ② Opposite angles are congruent.
- ③ Consecutive angles are supplementary.
- ④ Diagonals bisect each other.

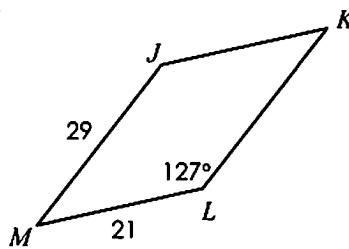
**Directions:** Each quadrilateral below is a parallelogram. Find the missing measures.

1.



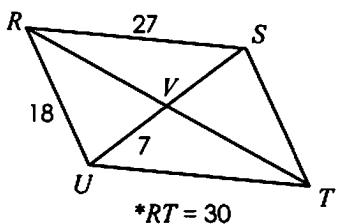
$$\begin{aligned}AD &= 8 \\DC &= 15 \\m\angle A &= 112^\circ \\m\angle B &= 68^\circ \\m\angle C &= 112^\circ\end{aligned}$$

2.



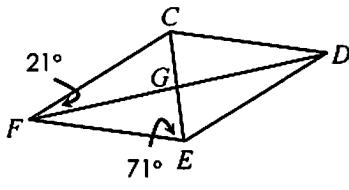
$$\begin{aligned}JK &= 21 \\KL &= 29 \\m\angle J &= 127^\circ \\m\angle K &= 53^\circ \\m\angle M &= 53^\circ\end{aligned}$$

3.



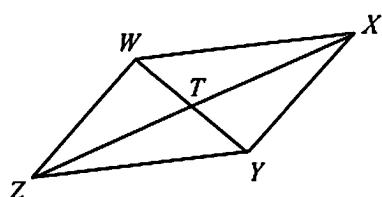
$$\begin{aligned}UT &= 27 \\ST &= 18 \\VS &= 7 \\VT &= 15\end{aligned}$$

4.



$$\begin{aligned}m\angle DEC &= 63^\circ \\m\angle CDE &= 46^\circ \\m\angle ECD &= 71^\circ \\m\angle DFE &= 25^\circ\end{aligned}$$

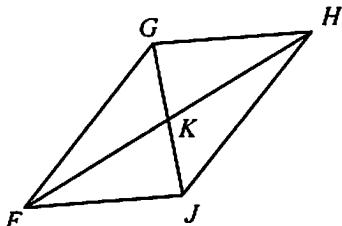
5. Given  $XY = 15$ ,  $WX = 22$ ,  $ZX = 32$ ,  $WT = 10$ ,  $m\angle WZY = 62^\circ$ ,  $m\angle WXT = 27^\circ$ , and  $m\angle ZWT = 77^\circ$ .



$$\begin{aligned}ZW &= 15 & m\angle TZY &= 27^\circ \\ZY &= 22 & m\angle XYZ &= 118^\circ \\TX &= 16 & m\angle XWT &= 41^\circ \\WY &= 20 & m\angle XYT &= 77^\circ\end{aligned}$$

★ Alt. Int. L's

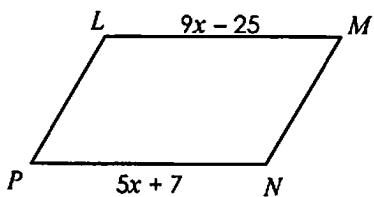
6. Given  $m\angle GHF = 34^\circ$ ,  $m\angle HJF = 124^\circ$ , and  $m\angle FKJ = 79^\circ$ .



★ Alt. Int. L's

$$\begin{aligned}m\angle GFJ &= 56^\circ & m\angle JGH &= 67^\circ \\m\angle FGH &= 124^\circ & m\angle FGJ &= 57^\circ \\m\angle HFJ &= 34^\circ & m\angle FHJ &= 22^\circ \\m\angle HKJ &= 101^\circ & m\angle GJF &= 67^\circ\end{aligned}$$

7. Solve for  $x$ .

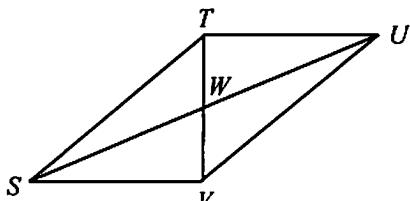


$$9x - 25 = 5x + 7$$

$$4x = 32$$

$$\boxed{x = 8}$$

9. If  $TV = 74$  and  $WV = 4x + 1$ , solve for  $x$ .



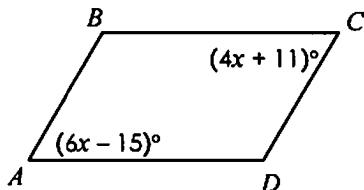
$$2(4x+1) = 74$$

$$8x + 2 = 74$$

$$8x = 72$$

$$\boxed{x = 9}$$

11. Find  $m\angle B$ .



$$6x - 15 = 4x + 11$$

$$2x = 26$$

$$x = 13$$

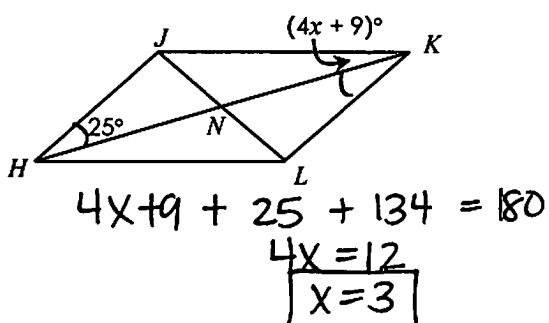
$$m\angle C = 4(13) + 11$$

$$= 63$$

$$m\angle B = 180 - 63$$

$$= 117^\circ$$

13. If  $m\angle KHL = 134^\circ$ , solve for  $x$ .

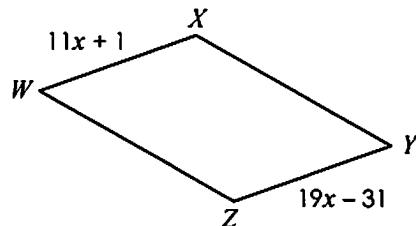


$$4x + 9 + 25 + 134 = 180$$

$$4x = 12$$

$$\boxed{x = 3}$$

8. Find  $YZ$ .



$$11x + 1 = 19x - 31$$

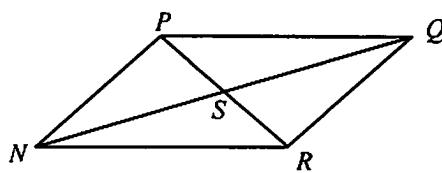
$$32 = 8x$$

$$x = 4$$

$$YZ: 19(4) - 31$$

$$= 45$$

10. If  $NS = 2x + 7$  and  $SQ = 5x - 23$ , find  $NQ$ .



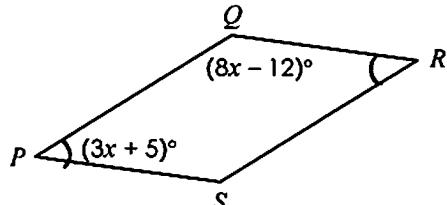
$$5x - 23 = 2x + 7$$

$$3x = 30$$

$$x = 10$$

$$NS = 2(10) + 7 = 27 ; \boxed{NQ = 54}$$

12. Find  $m\angle R$ .



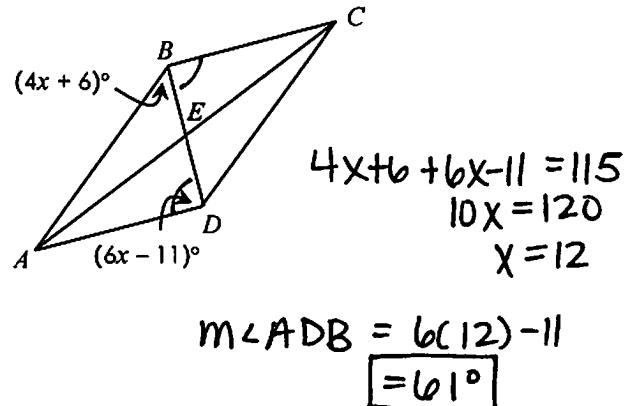
$$8x - 12 + 3x + 5 = 180$$

$$11x = 187$$

$$x = 17$$

$$m\angle R = 3(17) + 5 = \boxed{56^\circ}$$

14. If  $m\angle ABC = 115^\circ$ , find  $m\angle ADB$ .



$$4x + 6 + 6x - 11 = 115$$

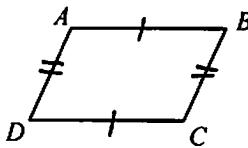
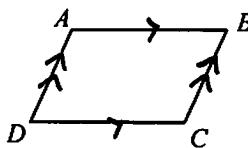
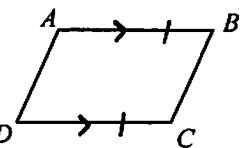
$$10x = 120$$

$$x = 12$$

$$m\angle ADB = 6(12) - 11$$

$$= 61^\circ$$

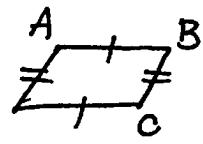
# PROVING PARALLELOGRAMS in the Coordinate Plane

<b>METHOD 1</b>  <p>Prove both pairs of opposite sides are congruent.</p>	<p>If <math>\overline{AB} \cong \overline{CD}</math> and <math>\overline{DA} \cong \overline{BC}</math>, then <math>ABCD</math> is a parallelogram.</p>	<p>Use.... <b>distance formula</b></p>
<b>METHOD 2</b>  <p>Prove both pairs of opposite sides are parallel.</p>	<p>If <math>\overline{AB} \parallel \overline{CD}</math> and <math>\overline{AD} \parallel \overline{BC}</math>, then <math>ABCD</math> is a parallelogram.</p>	<p>Use.... <b>slope formula</b></p>
<b>METHOD 3</b>  <p>Prove one pair of opposite sides are congruent and parallel.</p>	<p>If <math>\overline{AB} \cong \overline{CD}</math> and <math>\overline{AB} \parallel \overline{CD}</math>, then <math>ABCD</math> is a parallelogram.</p>	<p>Use.... <b>distance + slope formulas</b></p>

**SET I:** Use the distance formula to determine if the figure is a parallelogram.

1.  $A(-7, 4)$ ,  $B(1, 2)$ ,  $C(9, -8)$ ,  $D(1, -6)$

$$\begin{aligned} AB &= \sqrt{(1+7)^2 + (2-4)^2} = \sqrt{64+4} = \sqrt{68} = 2\sqrt{17} \\ BC &= \sqrt{(9-1)^2 + (-8-2)^2} = \sqrt{64+100} = \sqrt{164} = 2\sqrt{41} \\ CD &= \sqrt{(1-9)^2 + (-6+8)^2} = \sqrt{64+4} = \sqrt{68} = 2\sqrt{17} \\ DA &= \sqrt{(1+7)^2 + (-6-4)^2} = \sqrt{64+100} = \sqrt{164} = 2\sqrt{41} \end{aligned}$$



$$\overline{AB} \cong \overline{CD}$$

$$\overline{BC} \cong \overline{DA}$$

Yes!  $ABCD$  is a parallelogram.

2.  $P(-4, 2)$ ,  $Q(6, 4)$ ,  $R(11, -2)$ ,  $S(2, -3)$

$$PQ = \sqrt{(6+4)^2 + (4-2)^2} = \sqrt{100+4} = \sqrt{104} = 2\sqrt{26}$$

$$QR = \sqrt{(11-6)^2 + (-2-4)^2} = \sqrt{25+36} = \sqrt{61}$$

$$RS = \sqrt{(2-11)^2 + (-3+2)^2} = \sqrt{81+1} = \sqrt{82}$$

$$SP = \sqrt{(2+4)^2 + (-3-2)^2} = \sqrt{36+25} = \sqrt{61}$$

$$PQ \neq RS$$

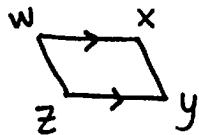
No!  $PQRS$  is not a parallelogram.

**SET 2:** Use the slope formula to determine if the figure is a parallelogram.

3.  $W(-7, -4), X(1, -6), Y(5, -13), Z(1, -12)$

$$\begin{aligned} m \overline{WX} &: \frac{-6+4}{1+7} = \frac{-2}{8} = -\frac{1}{4} \\ m \overline{XY} &: \frac{-13+6}{5-1} = \frac{-7}{4} \\ m \overline{YZ} &: \frac{-12+13}{1-5} = -\frac{1}{4} \\ m \overline{ZW} &: \frac{-12+4}{1+7} = \frac{-8}{8} = -1 \end{aligned}$$

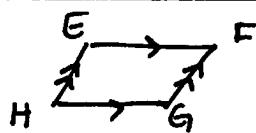
$$\overline{WX} \parallel \overline{YZ}$$



No!  $WXYZ$  is not a parallelogram.

4.  $E(0, 8), F(6, 10), G(2, 0), H(-4, -2)$

$$\begin{aligned} m \overline{EF} &: \frac{10-8}{6-0} = \frac{2}{6} = \frac{1}{3} \\ m \overline{FG} &: \frac{0-10}{2-6} = \frac{-10}{-4} = \frac{5}{2} \\ m \overline{GH} &: \frac{-2-0}{-4-2} = \frac{-2}{-6} = \frac{1}{3} \\ m \overline{HE} &: \frac{-2-8}{-4-0} = \frac{-10}{-4} = \frac{5}{2} \end{aligned}$$



$$\begin{aligned} \overline{EF} &\parallel \overline{GH} \\ \overline{FG} &\parallel \overline{HE} \end{aligned}$$

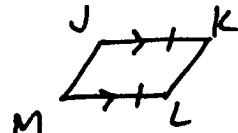
Yes!  $EFGH$  is a parallelogram.

**SET 3:** Use the distance formula AND slope formula to determine if the figure is a parallelogram.

5.  $J(-9, -2), K(-5, 1), L(1, -4), M(-3, -7)$

$$JK: d = \sqrt{(-5+9)^2 + (1+2)^2} = \sqrt{16+9} = \sqrt{25} = 5$$

$$LM: d = \sqrt{(-3-1)^2 + (-7+4)^2} = \sqrt{16+9} = \sqrt{25} = 5$$



$$m \overline{JK}: \frac{1+2}{-5+9} = \frac{3}{4}$$

$$\begin{aligned} \overline{JK} &\cong \overline{LM} \\ \overline{JK} &\parallel \overline{LM} \end{aligned}$$

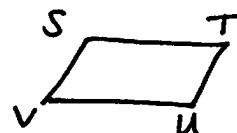
$$m \overline{LM}: \frac{-7+4}{-3-1} = \frac{-3}{-4} = \frac{3}{4}$$

Yes!  $JKLM$  is a parallelogram.

6.  $S(1, 5), T(10, 7), U(14, 1), V(-3, -1)$

$$ST: d = \sqrt{(10-1)^2 + (7-5)^2} = \sqrt{81+4} = \sqrt{85}$$

$$UV: d = \sqrt{(-3-14)^2 + (-1-1)^2} = \sqrt{289+4} = \sqrt{293}$$



$$m \overline{ST}: \frac{7-5}{10-1} = \frac{2}{9}$$

No!  $STUV$  is not a parallelogram.

$$m \overline{UV}: \frac{-1-1}{-3-14} = \frac{-2}{-17} = \frac{2}{17}$$

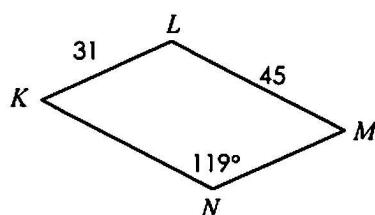
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**Unit 8: Polygons & Quadrilaterals**

Date: \_\_\_\_\_ Per: \_\_\_\_\_

**Homework 2: Parallelograms****\*\* This is a 2-page document! \*\*****Directions:** If each quadrilateral below is a parallelogram, find the missing measures.

1.



$MN = 31$

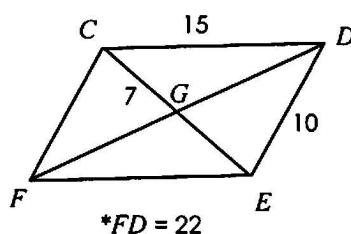
$KN = 45$

$m\angle K = 61^\circ$

$m\angle L = 119^\circ$

$m\angle M = 61^\circ$

2.

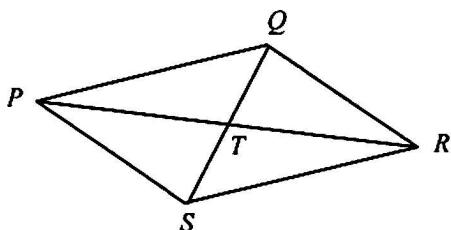


$CF = 10$

$FE = 15$

$CE = 14$

$GD = 11$

3. Given  $PQ = 24$ ,  $PS = 19$ ,  $PR = 42$ ,  $TQ = 10$ ,  $m\angle PQR = 106^\circ$ ,  $m\angle QSR = 49^\circ$ , and  $m\angle PRS = 35^\circ$ .

$QR = 19$

$SR = 24$

$PT = 21$

$SQ = 20$

$m\angle QRS = 74^\circ$

$m\angle PQS = 49^\circ$

$m\angle RPS = 39^\circ$

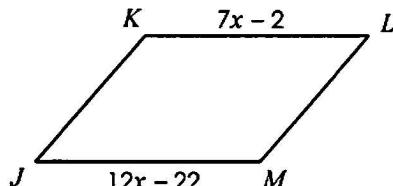
$m\angle PSQ = 57^\circ$

4. Find  $KL$ .

$12x - 22 = 7x - 2$

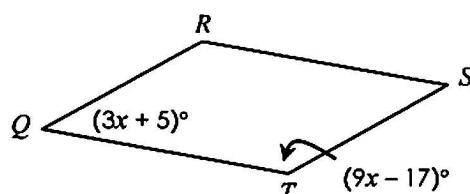
$5x = 20$

$x = 4$



$KL = 7(4) - 2$

$= 26$

6. Solve for  $x$ .

$3x + 5 + 9x - 17 = 180$

$12x - 12 = 180$

$12x = 192$

$x = 16$

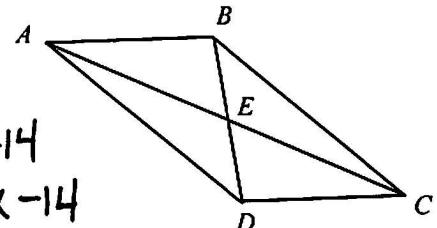
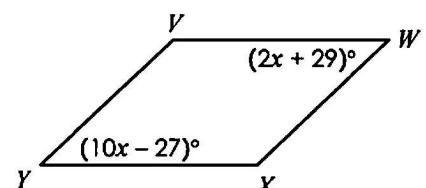
5. If  $AC = 8x - 14$  and  $EC = 2x + 11$ , solve for  $x$ .

$2(2x + 11) = 8x - 14$

$4x + 22 = 8x - 14$

$36 = 4x$

$x = 9$

7. Find  $m\angle V$ .

$10x - 27 = 2x + 29$

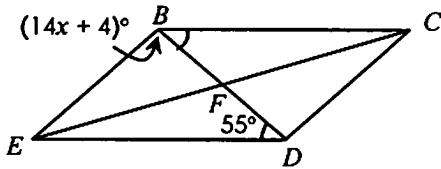
$8x = 56$

$x = 7$

$m\angle Y = 10(7) - 27 = 43^\circ$

$m\angle V = 137^\circ$

8. If  $m\angle BCD = 51^\circ$ , solve for  $x$ .



$$14x + 4 + 51 + 55 = 180$$

$$14x + 110 = 180$$

$$14x = 70$$

$$\boxed{x=5}$$

9. If  $m\angle VST = (5x + 23)^\circ$  and  $m\angle VUT = (8x - 49)^\circ$ , find  $m\angle SVT$ .

$$8x - 49 = 5x + 23$$

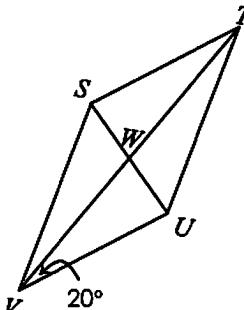
$$3x = 72$$

$$x = 24$$

$$m\angle VST = 5(24) + 23$$

$$= 143^\circ$$

$$m\angle SVT = \boxed{17^\circ}$$



**Directions:** Determine whether the quadrilateral is a parallelogram using the indicated method.

10.  $Q(-10, -2)$ ,  $R(1, -1)$ ,  $S(1, -7)$ ,  $T(-11, -8)$  (Distance Formula)

$$QR : d = \sqrt{(-10-1)^2 + (-2+1)^2} = \sqrt{121+1} = \sqrt{122}$$

$$ST : d = \sqrt{(1+11)^2 + (-7+8)^2} = \sqrt{144+1} = \sqrt{145}$$

No! QRST is not a parallelogram.

11.  $K(2, 7)$ ,  $L(6, 12)$ ,  $M(13, 13)$ ,  $N(9, 8)$  (Slope Formula)

$$m \overline{KL} : \frac{12-7}{6-2} = \frac{5}{4}$$

$$m \overline{MN} : \frac{8-13}{9-13} = \frac{-5}{-4} = \frac{5}{4}$$

$$m \overline{LM} : \frac{13-12}{13-6} = \frac{1}{7}$$

$$m \overline{KN} : \frac{8-7}{9-2} = \frac{1}{7}$$

Yes! KLMN is a parallelogram.

12.  $D(-5, -6)$ ,  $E(5, 2)$ ,  $F(4, -4)$ ,  $G(-6, -12)$  (Distance & Slope Formulas)

$$DE : d = \sqrt{(-5-5)^2 + (-6-2)^2} = \sqrt{100+64} = \sqrt{164} = 2\sqrt{41}$$

$$FG : d = \sqrt{(4+6)^2 + (-4+12)^2} = \sqrt{100+64} = \sqrt{164} = 2\sqrt{41}$$

$$m \overline{DE} : \frac{2+6}{5+5} = \frac{8}{10} = \frac{4}{5}$$

$$m \overline{FG} : \frac{-12+4}{-6-4} = \frac{-8}{-10} = \frac{4}{5}$$

Yes! DEFG is a parallelogram.

# PARALLELOGRAM Proofs

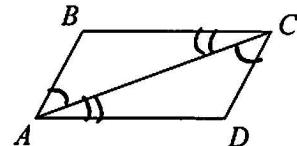
Recall: What is the definition of a parallelogram?

A quadrilateral in which both pairs of opposite sides are parallel.

Use the definition of a parallelogram to complete each proof.

- 1 Given:  $\angle BAC \cong \angle DCA$ ;  $\angle BCA \cong \angle DAC$

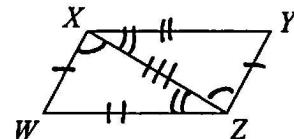
Prove: ABCD is a parallelogram



Statements	Reasons
1. $\angle BAC \cong \angle DCA$ ; $\angle BCA \cong \angle DAC$	1. given
2. $\overline{BA} \parallel \overline{DC}$ ; $\overline{BC} \parallel \overline{DA}$	2. Alt. Interior Angles Converse
3. ABCD is a parallelogram	3. Def. of parallelogram

- 2 Given:  $\overline{WX} \cong \overline{YZ}$ ;  $\overline{WZ} \cong \overline{YX}$

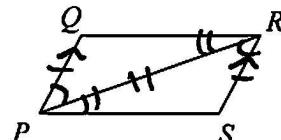
Prove: WXYZ is a parallelogram



Statements	Reasons
1. $\overline{WX} \cong \overline{YZ}$ ; $\overline{WZ} \cong \overline{YX}$	1. given
2. $\overline{XZ} \cong \overline{ZX}$	2. reflexive property
3. $\triangle WZX \cong \triangle YXZ$	3. SSS Congruence
4. $\angle WZX \cong \angle YZX$ ; $\angle WZC \cong \angle YXC$	4. CPCTC
5. $\overline{WX} \parallel \overline{YZ}$ ; $\overline{WZ} \parallel \overline{YX}$	5. Alt. Interior Angles Converse
6. WXYZ is a parallelogram	6. Def of parallelogram

- 3 Given:  $\overline{PQ} \cong \overline{RS}$ ;  $\overline{PQ} \parallel \overline{RS}$

Prove: PQRS is a parallelogram



Statements	Reasons
1. $\overline{PQ} \cong \overline{RS}$ ; $\overline{PQ} \parallel \overline{RS}$	1. given
2. $\angle QPR \cong \angle SPR$	2. Alt. Interior Angles
3. $\overline{PR} \cong \overline{RP}$	3. Reflexive Property
4. $\triangle QPR \cong \triangle SPR$	4. SAS Congruence
5. $\angle QRP \cong \angle SPR$	5. CPCTC
6. $\overline{QR} \parallel \overline{SP}$	6. Alt. Interior Angles Converse
7. PQRS is a parallelogram	7. Def of parallelogram

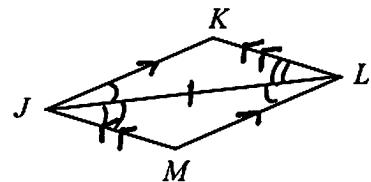
**Other properties of parallelograms:**

- Opposite sides are congruent.
- Opposite angles are congruent.
- Diagonals bisect each other.
- Adjacent angles are supplementary.

Complete each proof.

4 Given:  $JKLM$  is a parallelogram

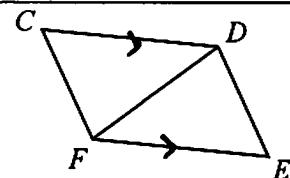
Prove:  $\overline{JK} \cong \overline{LM}$  and  $\overline{JM} \cong \overline{LK}$



Statements	Reasons
1. $JKLM$ is a parallelogram	1. given
2. $\overline{JK} \parallel \overline{LM}$ ; $\overline{JM} \parallel \overline{KL}$	2. Def. of parallelogram
3. $\angle KJL \cong \angle MLL$ ; $\angle MLJ \cong \angle KJL$	3. Alt. Interior Angles
4. $\overline{JL} \cong \overline{LJ}$	4. Reflexive Property
5. $\triangle JKJ \cong \triangle LMJ$	5. ASA Congruence
6. $\overline{JK} \cong \overline{LM}$ and $\overline{JM} \cong \overline{LK}$	6. CPCTC

5 Given:  $CDEF$  is a parallelogram

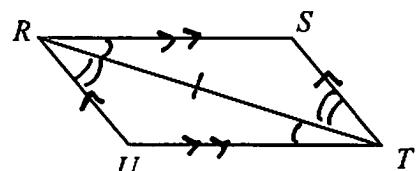
Prove:  $\angle DCF$  and  $\angle CFE$  are supplementary



Statements	Reasons
1. $CDEF$ is a parallelogram	1. given
2. $\overline{CD} \parallel \overline{EF}$	2. Def of Parallelogram
3. $\angle DCF$ and $\angle CFE$ are supplementary	3. Consec. Interior Angles

6 Given:  $RSTU$  is a parallelogram

Prove:  $\angle U \cong \angle S$



Statements	Reasons
1. $RSTU$ is a parallelogram	1. given
2. $\overline{RU} \parallel \overline{TS}$ ; $\overline{RS} \parallel \overline{TU}$	2. Def of parallelogram
3. $\angle SRT \cong \angle UTR$ ; $\angle URT \cong \angle STR$	3. Alt. Interior Angles
4. $\overline{RT} \cong \overline{TR}$	4. Reflexive Property
5. $\triangle RUT \cong \triangle TSR$	5. ASA Congruence
6. $\angle U \cong \angle S$	6. CPCTC

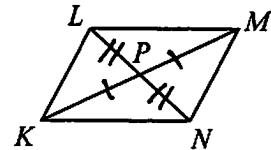
**Properties of parallelogram are sufficient to prove a quadrilateral is a parallelogram.**  
 The list below summarizes ways to prove a quadrilateral is a parallelogram.

- Show that both pairs of opposite sides are parallel. (Definition of a Parallelogram)
- Show that both pairs of opposite sides are congruent.
- Show that both pairs of opposite angles are congruent.
- Show that the diagonals bisect each other.
- Show that one angle is supplementary to both adjacent angles.
- Show that one pair of opposite sides are congruent and parallel.

Complete each proof.

7 Given:  $P$  is the midpoint of  $\overline{LN}$  and  $\overline{KM}$

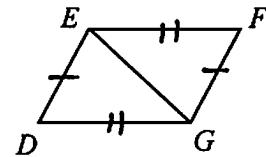
Prove:  $KLMN$  is a parallelogram



Statements	Reasons
1. $P$ is the midpoint of $\overline{LN}$ & $\overline{KM}$	1. given
2. $\overline{KP} \cong \overline{MP}$ ; $\overline{LP} \cong \overline{NP}$	2. Def of midpoint
3. $KLMN$ is a parallelogram	3. Diag. of p'gram bisect each other

8 Given:  $\triangle EDG \cong \triangle GFE$

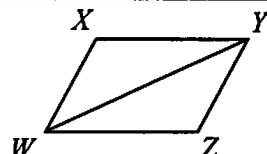
Prove:  $DEFG$  is a parallelogram



Statements	Reasons
1. $\triangle EDG \cong \triangle GFE$	1. given
2. $\overline{DG} \cong \overline{FE}$ ; $\overline{DE} \cong \overline{FG}$	2. CPCTC
3. $DEFG$ is a parallelogram	3. Opp. sides of p'gram are congruent

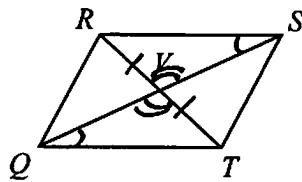
9 Given:  $\angle XWZ \cong \angle ZYX$ ;  $\triangle WXY \cong \triangle YZW$

Prove:  $WXYZ$  is a parallelogram



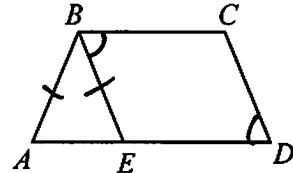
Statements	Reasons
1. $\angle XWZ \cong \angle ZYX$	1. given
2. $\triangle WXY \cong \triangle YZW$	2. given
3. $\angle WXY \cong \angle YZW$	3. CPCTC
4. $WXYZ$ is a parallelogram	4. Opp. angles of p'gram are congruent.

- 10 Given:  $\overline{QS}$  bisects  $\overline{RT}$ ;  $\angle RSQ \cong \angle TQS$   
 Prove:  $QRST$  is a parallelogram



Statements	Reasons
1. $\overline{QS}$ bisects $\overline{RT}$ ; $\angle RSQ \cong \angle TQS$	1. given
2. $\overline{RV} \cong \overline{TV}$	2. Def of bisector
3. $\angle RVS \cong \angle TVQ$	3. Vertical angles
4. $\triangle RVS \cong \triangle TVQ$	4. AAS $\triangle$ Congruence
5. $RS \cong QT$	5. CPCTC
6. $RS \parallel QT$	6. Alt. Interior Angles Converse
7. $QRST$ is a parallelogram	7. One pair of opp sides are congruent + parallel

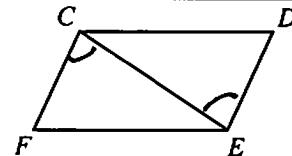
- 11 Given:  $\overline{BA} \cong \overline{BE}$ ,  $\angle CBE \cong \angle EDC$ ,  
 $\angle A$  and  $\angle C$  are supplementary  
 Prove:  $BCDE$  is a parallelogram



Statements	Reasons
1. $\overline{BA} \cong \overline{BE}$ ; $\angle CBE \cong \angle EDC$	1. given
2. $\triangle ABE$ is an isosceles $\triangle$	2. Def of isosceles triangle
3. $\angle A \cong \angle BEA$	3. Base $\angle$ 's of isosc $\triangle$ are congruent
4. $m\angle A = m\angle BEA$	4. Def of congruence
5. $\angle A + \angle C$ are supplementary	5. given
6. $\angle BEA + \angle BED$ are supplementary	6. Linear Pair (Supplement Thm)
7. $m\angle A + m\angle C = 180^\circ$	7. Def of supplementary angles
8. $m\angle BEA + m\angle BED = 180^\circ$	8. Def of supplementary angles
9. $m\angle BEA + m\angle BED = m\angle A + m\angle C$	9. Substitution Property
10. $m\angle A + m\angle BED = m\angle A + m\angle C$	10. Substitution Property
11. $m\angle BED = m\angle C$	11. Subtraction Property
12. $\angle BED \cong \angle C$	12. Def of congruence
13. $BCDE$ is a parallelogram	13. Opp. angles of a p'gram are congruent.

12 Given:  $\angle FCE \cong \angle DEC$ ,  $\angle FCD \cong \angle DEF$

Prove:  $CDEF$  is a parallelogram

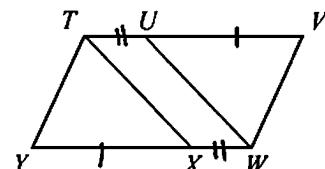


Statements	Reasons
1. $\angle FCE \cong \angle DEC$	1. given
2. $\overline{FC} \parallel \overline{DE}$	2. Alt. Interior Angles Converse
3. $\angle FCD \cong \angle DEF$	3. given
4. $m\angle FCE + m\angle DCE = m\angle FCD$	4. Angle Addition Postulate
5. $m\angle FEC + m\angle DEC = m\angle FED$	5. Angle Addition Postulate
6. $m\angle FCE = m\angle DEC$	6. Def of Congruence
7. $m\angle FCD = m\angle DEF$	7. Def of Congruence
8. $m\angle DEC + m\angle DCE = m\angle DEF$	8. Substitution Property
9. $m\angle FEC + m\angle DEC = m\angle DEC + m\angle DCE$	9. Substitution Property
10. $m\angle FEC = m\angle DCE$	10. Subtraction Property
11. $\overline{CD} \parallel \overline{FE}$	11. Alt. Interior Angles Converse
12. $CDEF$ is a parallelogram	12. Def of parallelogram

13 Given:  $TUWX$  is a parallelogram,

$$\overline{YX} \cong \overline{UV}$$

Prove:  $TVWY$  is a parallelogram

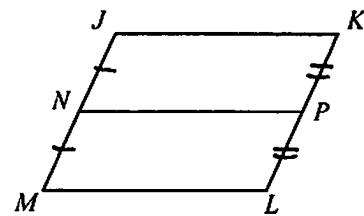


Statements	Reasons
1. $TUWX$ is a parallelogram.	1. given
2. $\overline{TU} \parallel \overline{WX}$	2. Def of a parallelogram
3. $\overline{TU} \cong \overline{WX}$	3. Opp sides of a p'gram are congruent.
4. $\overline{YX} \cong \overline{UV}$	4. given
5. $UV + TU = TV$	5. Segment Addition Postulate
6. $YX + XW = YW$	6. Segment Addition Postulate
7. $TU = XW$ ; $YX = UV$	7. Def of Congruence
8. $UV + TU = YW$	8. Substitution Property
9. $TV = YW$	9. Substitution Property
10. $\overline{TV} \cong \overline{YW}$	10. Def of Congruence
11. $TVWY$ is a parallelogram	11. One pair of opp. sides are parallel + congruent.

14

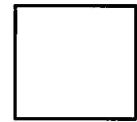
**Given:**  $JKLM$  is a parallelogram,  
 $N$  is the midpoint of  $\overline{MJ}$ ,  
 $P$  is the midpoint of  $\overline{KL}$

**Prove:**  $JKPN$  is a parallelogram

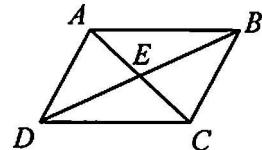


Statements	Reasons
1. $JKLM$ is a parallelogram	1. given
2. $N$ is the midpoint of $\overline{MJ}$	2. given
3. $P$ is the midpoint of $\overline{KL}$	3. given
4. $\overline{JN} \parallel \overline{KP}$	4. Def of parallelogram
5. $\overline{MN} \cong \overline{KL}$	5. Opp. sides of a p'gram are congruent
6. $JN = MN; KP = LP$	6. Def of midpoint
7. $JN + MN = MJ$	7. Seg Addition Postulate
8. $KP + LP = KL$	8. Seg Addition Postulate
9. $MJ = KL$	9. Def of Congruence
10. $JN + JN = MJ$	10. Substitution Property
11. $KP + KP = KL$	11. Substitution Property
12. $2JN = MJ; 2KP = KL$	12. Simplify
13. $2KP = MJ$	13. Substitution Property
14. $2JN = 2KP$	14. Substitution Property
15. $JN = KP$	15. Division Property
16. $\overline{JN} \cong \overline{KP}$	16. Def of Congruence
17. $JKPN$ is a parallelogram	17. One pair of opp. sides are parallel & congruent.

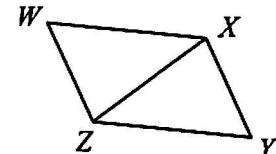
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**Unit 8: Polygons & Quadrilaterals**

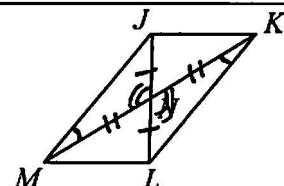
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**Homework 3: Parallelogram Proofs****\*\* This is a 2-page document! \*\*****Directions:** Complete each proof.**1. Given:**  $\overline{AD} \parallel \overline{BC}$ ;  $\angle ABD \cong \angle CDB$ **Prove:**  $ABCD$  is a parallelogram

Statements	Reasons
1. $\overline{AD} \parallel \overline{BC}$	1. given
2. $\angle ABD \cong \angle CDB$	2. given
3. $\overline{AB} \parallel \overline{CD}$	3. Alt. Interior Angles Converse
4. $ABCD$ is a parallelogram	4. Def of parallelogram

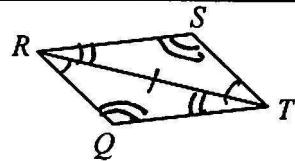
**2. Given:**  $\overline{WX} \cong \overline{YZ}$ ;  $\angle WXZ \cong \angle YZX$ **Prove:**  $WXYZ$  is a parallelogram

Statements	Reasons
1. $\overline{WX} \cong \overline{YZ}$ ; $\angle WXZ \cong \angle YZX$	1. given
2. $\overline{XZ} \cong \overline{ZX}$	2. Reflexive Property
3. $\triangle WXZ \cong \triangle YZX$	3. SAS Δ Congruence
4. $\overline{WZ} \cong \overline{YX}$	4. CPCTC
5. $WXYZ$ is a parallelogram	5. Opp sides of p'gram are congruent.

**3. Given:**  $\overline{JN} \cong \overline{NL}$ ;  $\angle JMK \cong \angle LKM$ **Prove:**  $JKLM$  is a parallelogram

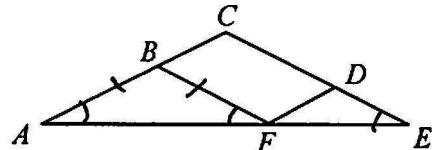
Statements	Reasons
1. $\overline{JN} \cong \overline{NL}$ ; $\angle JMK \cong \angle LKM$	1. given
2. $\angle JNM \cong \angle LNK$	2. Vertical Angles
3. $\triangle JNM \cong \triangle LNK$	3. AAS Δ congruence
4. $\overline{NM} \cong \overline{LK}$	4. CPCTC
5. $JKLM$ is a parallelogram	5. Diagonals of a p'gram bisect each other

4. Given:  $\angle QRT \cong \angle STR$ ;  $\angle S \cong \angle Q$   
 Prove:  $QRST$  is a parallelogram



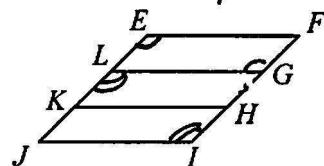
Statements	Reasons
1. $\angle QRT \cong \angle STR$ ; $\angle S \cong \angle Q$	1. given
2. $\overline{RT} \cong \overline{TR}$	2. Reflexive property
3. $\triangle RQT \cong \triangle TSR$	3. AAS $\triangle$ congruence
4. $\angle LSRT \cong \angle QTR$	4. CPCTC
5. $\overline{RQ} \parallel \overline{TS}$ ; $\overline{RS} \parallel \overline{TQ}$	5. Alt. Interior Angles Converse
6. $QRST$ is a parallelogram	6. Def of parallelogram

5. Given:  $BCDF$  is a parallelogram;  $\overline{AB} \cong \overline{BF}$   
 Prove:  $\angle A \cong \angle E$



Statements	Reasons
1. $BCDF$ is a parallelogram	1. given
2. $\overline{CD} \parallel \overline{BF}$	2. Def of parallelogram
3. $\angle BFA \cong \angle E$	3. Corresponding angles
4. $\overline{AB} \cong \overline{BF}$	4. given
5. $\triangle ABF$ is isosceles	5. Def of isosceles triangle
6. $\angle A \cong \angle BFA$	6. Base $\angle$ 's of isos $\triangle$ are congruent
7. $\angle A \cong \angle E$	7. Transitive Property

6. Given:  $EFGL$  and  $LGIJ$  are parallelograms  
 Prove:  $\angle E \cong \angle I$



Statements	Reasons
1. $EFGL$ & $LGIJ$ are parallelograms	1. given
2. $\angle E \cong \angle LGF$ ; $\angle JLG \cong \angle I$	2. Opp. angles of a $\square$ are congruent
3. $\overline{EF} \parallel \overline{LG}$	3. Def of parallelogram
4. $\angle E \cong \angle JLG$	4. Corresponding angles
5. $\angle E \cong \angle I$	5. Transitive Property

Name: \_\_\_\_\_

**Geometry**

Date: \_\_\_\_\_ Per: \_\_\_\_\_

**Unit 8: Polygons & Quadrilaterals****Quiz 8-1: Angles of Polygons & Parallelograms****Part I: Angles of Polygons**

1. What is the sum of the degrees of the interior angles of a 19-gon?

$$S = (19-2) \cdot 180$$

2. If the sum of the interior angles of a polygon is
- $1800^\circ$
- , how many sides does it have?

$$(n-2) \cdot 180 = 1800$$

$$n-2 = 10$$

3. What is the measure of an interior angle of a regular nonagon?

$$\frac{(9-2) \cdot 180}{9} = \frac{1260}{9}$$

4. What is the sum of the exterior angles of a 25-gon?

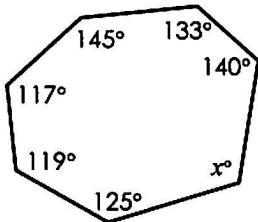
5. What is the measure of each exterior angle of a regular decagon?

$$\frac{360}{10}$$

Find the value of  $x$ .

6.  $x = 121^\circ$

$$(7-2) \cdot 180 = 900$$

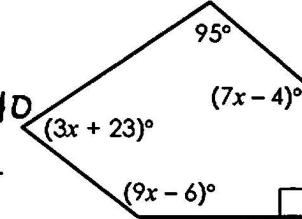


7.  $x = 18$

$$19x + 198 = 540$$

$$19x = 342$$

$$x = 18$$



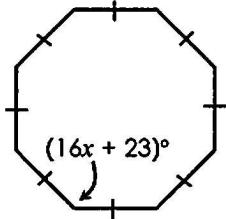
8.  $x = 7$

$$\frac{(8-2) \cdot 180}{8} = 135$$

$$16x + 23 = 135$$

$$16x = 112$$

$$x = 7$$

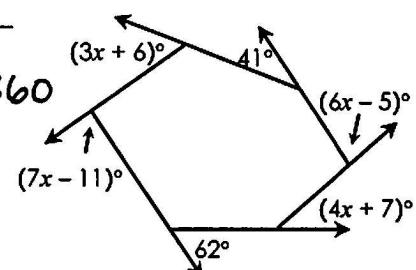


9.  $x = 13$

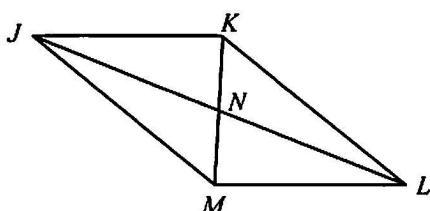
$$20x + 100 = 360$$

$$20x = 260$$

$$x = 13$$

**Part II: Parallelograms**

10. Given
- $JM = 27$
- ,
- $ML = 16$
- ,
- $JL = 46$
- ,
- $NK = 15$
- ,
- $m\angle KLM = 48^\circ$
- ,
- $m\angle JKM = 78^\circ$
- , and
- $m\angle MJL = 22^\circ$
- , find each missing value.



$$KL = 27$$

$$JK = 16$$

$$MK = 30$$

$$NL = 23$$

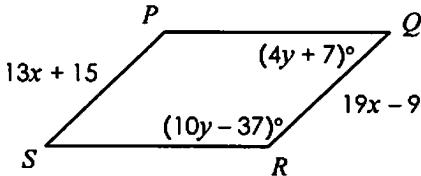
$$m\angle JKL = 132^\circ$$

$$m\angle KLJ = 22^\circ$$

$$m\angle KMJ = 54^\circ$$

$$m\angle KJL = 26^\circ$$

11. If  $PQRS$  is a parallelogram, find the values of  $x$  and  $y$ .

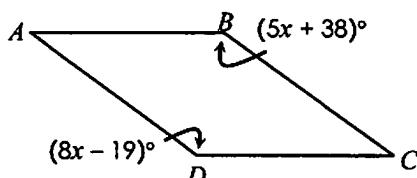


$$\begin{aligned} 19x - 9 &= 13x + 15 \\ 6x &= 24 \\ x &= 4 \end{aligned}$$

$$\begin{aligned} 14y - 80 &= 180 \\ 14y &= 260 \\ y &= 15 \end{aligned}$$

11.  $x = \underline{\hspace{2cm}} \quad 4$   
 $y = \underline{\hspace{2cm}} \quad 15$

12. If  $ABCD$  is a parallelogram, find  $m\angle C$ .



$$\begin{aligned} 8x - 19 &= 5x + 38 \\ 3x &= 57 \\ x &= 19 \end{aligned}$$

$$\begin{aligned} m\angle B &= 5(19) + 38 \\ &= 133^\circ \\ m\angle C &= 180 - 133 \\ &= 47^\circ \end{aligned}$$

Determine whether the quadrilateral is a parallelogram using the indicated method.

13.  $D(-8, 1), E(-3, 6), F(7, 4), G(2, -1)$  (Distance Formula)

13. YES / NO

$$\begin{aligned} DE: d &= \sqrt{(-3+8)^2 + (6-1)^2} = \sqrt{25+25} = \sqrt{50} = 5\sqrt{2} \\ FG: d &= \sqrt{(2-7)^2 + (-1-4)^2} = \sqrt{25+25} = \sqrt{50} = 5\sqrt{2} \\ EF: d &= \sqrt{(7+3)^2 + (4-6)^2} = \sqrt{100+4} = \sqrt{104} = 2\sqrt{26} \\ DG: d &= \sqrt{(2+8)^2 + (-1-1)^2} = \sqrt{100+4} = \sqrt{104} = 2\sqrt{26} \end{aligned} \quad \left. \begin{array}{l} \\ \\ \\ \end{array} \right\} \cong$$

14.  $L(-1, 6), M(5, 9), N(0, 2), P(-8, -2)$  (Slope Formula)

14. YES / NO

$$\begin{aligned} m \overline{LM} &: \frac{9-6}{5+1} = \frac{3}{6} = \frac{1}{2} \\ m \overline{NP} &: \frac{-2-2}{-8-0} = \frac{-4}{-8} = \frac{1}{2} \end{aligned} \quad \left. \begin{array}{l} \\ \end{array} \right\} \parallel$$

$$\begin{aligned} m \overline{MN} &: \frac{2-9}{0-5} = \frac{-7}{-5} = \frac{7}{5} \\ m \overline{LP} &: \frac{-2-6}{-8+1} = \frac{-8}{-7} = \frac{8}{7} \end{aligned} \quad \left. \begin{array}{l} \\ \end{array} \right\} \text{Not } \parallel$$

15.  $B(-2, -9), C(0, -5), D(6, -3), E(4, -7)$  (Distance and Slope Formulas)

15. YES / NO

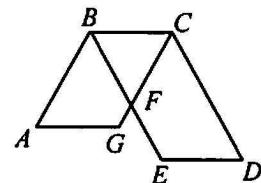
$$\begin{aligned} BC: d &= \sqrt{(0+2)^2 + (-5+9)^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5} \\ DE: d &= \sqrt{(4-6)^2 + (-7+3)^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5} \end{aligned} \quad \left. \begin{array}{l} \\ \end{array} \right\} \cong$$

$$\begin{aligned} m \overline{BC} &: \frac{-5+9}{0+2} = \frac{4}{2} = 2 \\ m \overline{DE} &: \frac{-7+3}{4-6} = \frac{-4}{-2} = 2 \end{aligned} \quad \left. \begin{array}{l} \\ \end{array} \right\} \parallel$$

Complete each proof.

16. Given:  $ABCG$  and  $BCDE$  are parallelograms

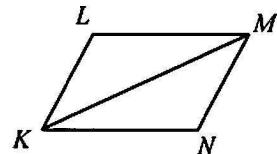
Prove:  $\overline{AG} \cong \overline{ED}$



Statements	Reasons
1. $ABCG$ & $BCDE$ are parallelograms	1. given
2. $\overline{AG} \cong \overline{BC}$ ; $\overline{BC} \cong \overline{ED}$	2. Opp sides of pgram are Congruent
3. $\overline{AG} \cong \overline{ED}$	3. Transitive property

17. Given:  $\overline{KL} \parallel \overline{NM}$ ;  $\angle L \cong \angle N$

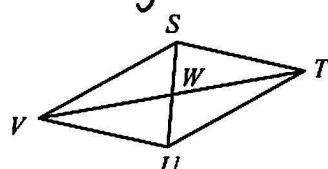
Prove:  $KLMN$  is a parallelogram



Statements	Reasons
1. $\overline{KL} \parallel \overline{NM}$	1. given
2. $\angle L \cong \angle N$	2. given
3. $\angle LKM \cong \angle NMK$	3. Alt. Interior Angles
4. $\overline{KM} \cong \overline{MK}$	4. Reflexive property
5. $\triangle LKM \cong \triangle NMK$	5. AAS $\triangle$ Congruence
6. $\triangle LMK \cong \triangle NKM$	6. CPCTC
7. $\overline{LM} \parallel \overline{NK}$	7. Alt. Interior Angles Converse
8. $KLMN$ is a parallelogram	8. Def of parallelogram

18. Given:  $W$  is the midpoint of  $\overline{SU}$ ;  $\overline{ST} \parallel \overline{VU}$

Prove:  $STUV$  is a parallelogram



Statements	Reasons
1. $W$ is the midpt of $\overline{SU}$	1. given
2. $\overline{SW} \cong \overline{UW}$	2. Def of midpoint
3. $\overline{ST} \parallel \overline{VU}$	3. given
4. $\angle TSU \cong \angle VUS$	4. Alt. Interior Angles
5. $\angle SWT \cong \angle UWV$	5. Vertical Angles
6. $\triangle SWT \cong \triangle UWV$	6. ASA $\triangle$ congruence
7. $\overline{ST} \cong \overline{VU}$	7. CPCTC
8. $STUV$ is a parallelogram	8. One pair of opp sides are $\cong$ and $\parallel$ .

Name:

Date:

Topic:

Class:

## Main Ideas/Questions

# Properties of RECTANGLES

PLUS THESE! ➤

**Rectangles have the same properties of parallelograms:**

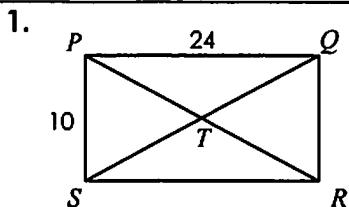
- Opposite sides are congruent.
- Opposite sides are parallel.
- Opposite angles are congruent.
- Consecutive angles are supplementary.
- Diagonals bisect each other.

①

All 4 angles are right angles.

②

Diagonals are congruent.

**Directions:** Each quadrilateral below is a rectangle. Find the missing measures.

$$QR = 10$$

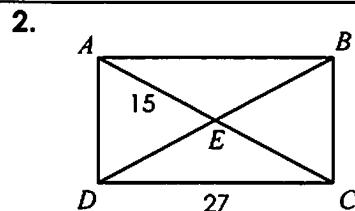
$$SR = 24$$

$$SQ = 26$$

$$PR = 26$$

$$QT = 13$$

$$\begin{aligned} SQ: 10^2 + 24^2 &= x^2 \\ 676 &= x^2 \\ x &= 26 \end{aligned}$$



$$AC = 36$$

$$BD = 30$$

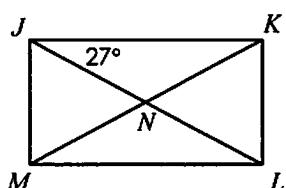
$$BE = 15$$

$$AB = 27$$

$$BC = 13.1$$

$$\begin{aligned} BC: 27^2 + x^2 &= 30^2 \\ x^2 &= 171 \\ x &= 13.1 \end{aligned}$$

3.



$$m\angle MJK = 90^\circ$$

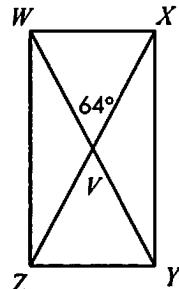
$$m\angle MJL = 63^\circ$$

$$m\angle JLK = 63^\circ$$

$$m\angle KML = 27^\circ$$

$$m\angle MNL = 126^\circ$$

4.



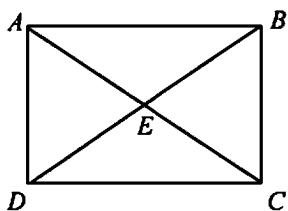
$$m\angle XWY = 58^\circ$$

$$m\angle YXZ = 32^\circ$$

$$m\angle WVZ = 116^\circ$$

$$m\angle XWZ = 90^\circ$$

$$m\angle XZY = 58^\circ$$

5. Given  $DB = 42$ ,  $AD = 26$ , and  $m\angle DAE = 52^\circ$ .

$$x^2 + 26^2 = 42^2$$

$$x^2 = 1088$$

$$x = 33$$

$$AC = 42$$

$$EB = 21$$

$$BC = 26$$

$$AB = 33$$

$$m\angle ADC = 90^\circ$$

$$m\angle ABD = 38^\circ$$

$$m\angle BCA = 52^\circ$$

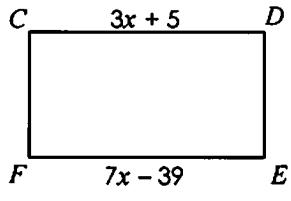
$$m\angle DEC = 104^\circ$$

6. Find  $EF$ .

$$7x - 39 = 3x + 5$$

$$4x = 44$$

$$x = 11$$



$$EF : 7(11) - 39 = \boxed{38}$$

8. If  $JM = x + 17$  and  $MK = 5x - 23$ , find  $JL$ .

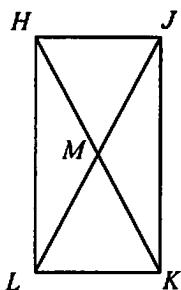
$$x + 17 = 5x - 23$$

$$40 = 4x$$

$$x = 10$$

$$JM : 10 + 17 = 27$$

$$\boxed{JL = 54}$$



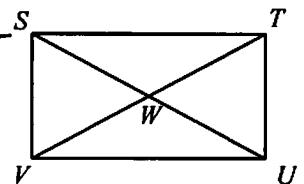
9. If  $VW = 9x - 11$  and  $SU = 16x - 12$ , find  $WT$ .

$$2(9x - 11) = 16x - 12$$

$$18x - 22 = 16x - 12$$

$$2x = 10$$

$$x = 5$$



$$VW : 9(5) - 11 = 34$$

$$\boxed{WT = 34}$$

10. Find  $m\angle BCE$ .

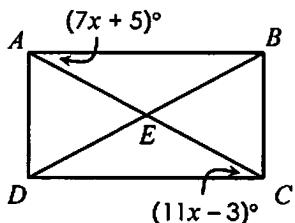
$$11x - 3 = 7x + 5$$

$$4x = 8$$

$$x = 2$$

$$m\angle DCE = 11(2) - 3 = 19^\circ$$

$$\boxed{m\angle BCE = 71^\circ}$$



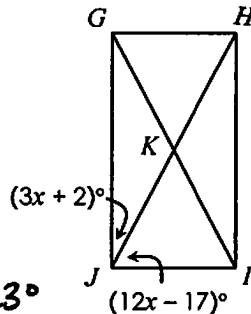
11. Find  $m\angle JHI$ .

$$3x + 2 + 12x - 17 = 90$$

$$15x - 15 = 90$$

$$15x = 105$$

$$x = 7$$



$$m\angle JIK = 3(7) + 2 = 23^\circ$$

$$\boxed{m\angle JHI = 23^\circ}$$

12. Find  $m\angle XZW$ .

$$5x - 8 + x + 20 = 90$$

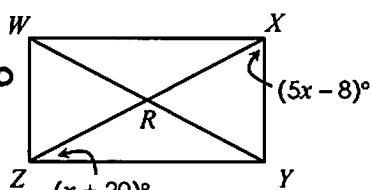
$$6x + 12 = 90$$

$$6x = 78$$

$$x = 13$$

$$m\angle YXZ = 5(13) - 8 = 57^\circ$$

$$\boxed{m\angle XZW = 57^\circ}$$



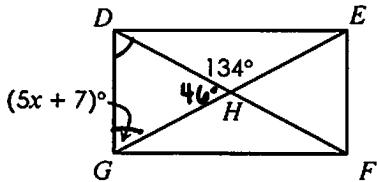
13. Solve for  $x$ .

$$2(5x + 7) + 46 = 180$$

$$10x + 14 + 46 = 180$$

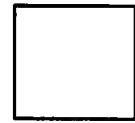
$$10x = 120$$

$$x = 12$$



Name: \_\_\_\_\_

## Unit 8: Polygons &amp; Quadrilaterals



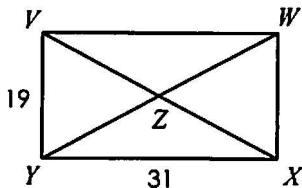
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## Homework 4: Rectangles

\*\* This is a 2-page document! \*\*

Directions: If each quadrilateral below is a rectangle, find the missing measures.

1.



$VW = 31$

$WX = 19$

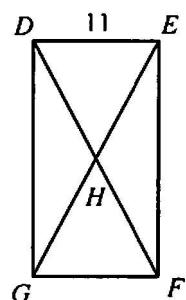
$YW = 36.4$

$ZX = 18.2$

$VX = 36.4$

$$\text{Why: } 31^2 + 19^2 = X^2 \\ 1322 = X^2 \\ X = 36.4$$

2.



$GF = 11$

$GE = 28$

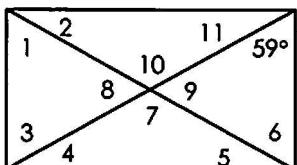
$DF = 28$

$HF = 14$

$DG = 25.7$

$$\text{DG: } X^2 + 11^2 = 28^2 \\ X^2 = 663 \\ X = 25.7$$

3.



$m\angle 1 = 59^\circ$

$m\angle 2 = 31^\circ$

$m\angle 3 = 59^\circ$

$m\angle 4 = 31^\circ$

$m\angle 5 = 31^\circ$

$m\angle 6 = 59^\circ$

$m\angle 7 = 118^\circ$

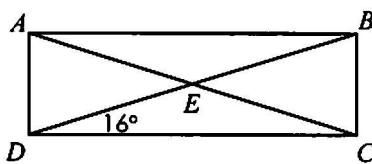
$m\angle 8 = 62^\circ$

$m\angle 9 = 62^\circ$

$m\angle 10 = 118^\circ$

$m\angle 11 = 31^\circ$

4.



$m\angle BCD = 90^\circ$

$m\angle ABD = 16^\circ$

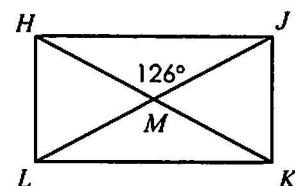
$m\angle CBE = 74^\circ$

$m\angle ADE = 74^\circ$

$m\angle AEB = 148^\circ$

$m\angle DEA = 32^\circ$

5.



$m\angle JMK = 54^\circ$

$m\angle JKH = 63^\circ$

$m\angle HLK = 90^\circ$

$m\angle HJL = 27^\circ$

$m\angle LHK = 63^\circ$

$m\angle JLK = 27^\circ$

6. Find WZ.

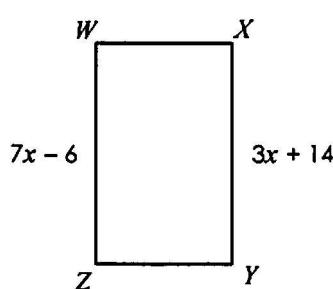
$7x - 6 = 3x + 14$

$4x = 20$

$x = 5$

$WZ: 7(5) - 6$

$= 29$

7. If  $SQ = 11x - 26$  and  $PR = 5x + 28$ , find  $PR$ .

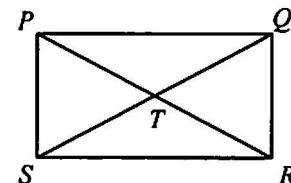
$11x - 26 = 5x + 28$

$6x = 54$

$x = 9$

$PR: 5(9) + 28$

$= 73$

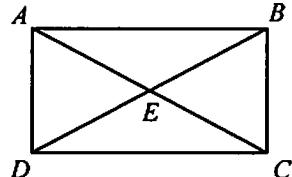


8. If  $AE = 6x - 55$  and  $EC = 3x - 16$ , find  $DB$ .

$$6x - 55 = 3x - 16$$

$$3x = 39$$

$$x = 13$$



$$EC : 3(13) - 16 = 23$$

$$AC : 2(23) = 46$$

$$\boxed{DB = 46}$$

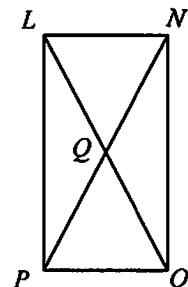
9. If  $LO = 15x + 19$  and  $QN = 10x + 2$ , find  $PN$ .

$$2(10x + 2) = 15x + 19$$

$$20x + 4 = 15x + 19$$

$$5x = 15$$

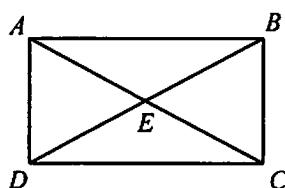
$$x = 3$$



$$LO : 15(3) + 19 = 64$$

$$\boxed{PN = 64}$$

10. If  $DE = 4x + 1$ ,  $EB = 12x - 31$ , and  $CD = 28$ , find  $AD$ .



$$12x - 31 = 4x + 1$$

$$8x = 32$$

$$x = 4$$

$$x^2 + 28^2 = 34^2$$

$$x^2 = 312$$

$$x = 19.3$$

$$DE : 4(4) + 1 = 17$$

$$DB = AC = 34$$

$$\boxed{AD = 19.3}$$

11. Find  $m\angle GJK$ .

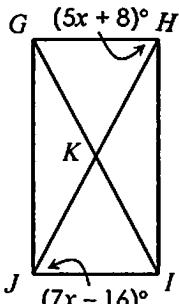
$$7x - 16 = 5x + 8$$

$$2x = 24$$

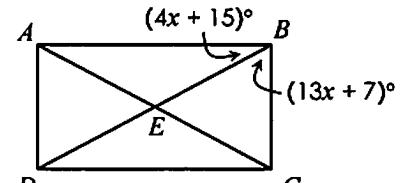
$$x = 12$$

$$m\angle HJI = 7(12) - 16 = 68^\circ$$

$$\boxed{m\angle GJK = 22^\circ}$$



12. Find  $m\angle ADE$ .



$$4x + 15 + 13x + 7 = 90$$

$$17x + 22 = 90$$

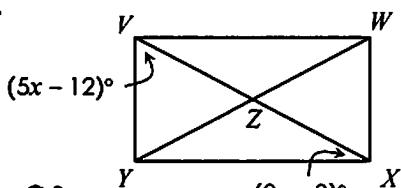
$$17x = 68$$

$$x = 4$$

$$m\angle CBD = 13(4) + 7 = 59^\circ$$

$$\boxed{m\angle ADE = 59^\circ}$$

13. Find  $m\angle VWZ$ .



$$5x - 12 + 2x - 3 = 90$$

$$7x - 15 = 90$$

$$7x = 105$$

$$x = 15$$

$$m\angle ZXY = 2(15) - 3 = 27^\circ$$

$$\boxed{m\angle VWZ = 27^\circ}$$

14. Find  $m\angle DHG$ .

$$14x - 27 = 9x + 3$$

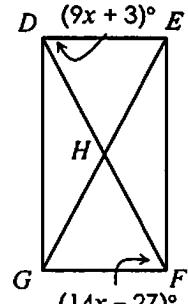
$$5x = 30$$

$$x = 6$$

$$m\angle EDH = 9(6) + 3 = 57^\circ$$

$$m\angle EHD = 180 - 2(57) = 66^\circ$$

$$\boxed{m\angle DHG = 114^\circ}$$

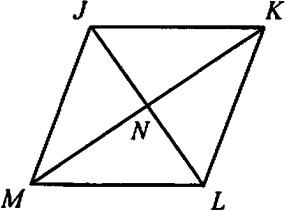
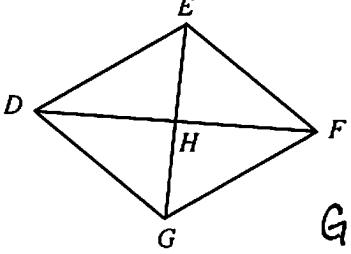
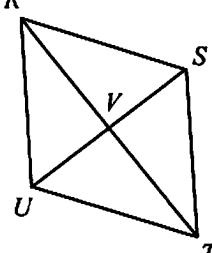
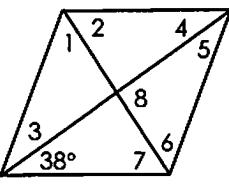
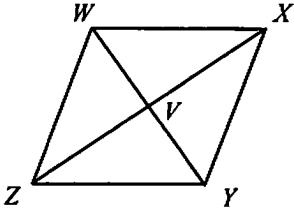


Name:

Date:

Topic:

Class:

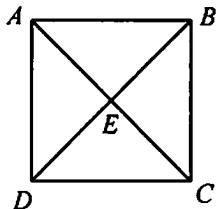
Main Ideas/Questions	Notes/Examples
<p><i>Properties of RHOMBI</i></p> <p>Plus these! ►</p>	<p><b>Rhombi have the same properties of parallelograms:</b></p> <ul style="list-style-type: none"> <li>Opposite sides are congruent.</li> <li>Opposite sides are parallel.</li> <li>Opposite angles are congruent.</li> <li>Consecutive angles are supplementary.</li> <li>Diagonals bisect each other.</li> </ul> <p>① All 4 sides are congruent.</p> <p>② Diagonals are perpendicular.</p> <p>③ Diagonals bisect opposite angles.</p>
<p><b>Directions:</b> Each quadrilateral below is a rhombus. Find the missing measures.</p> <p>1. <math>JK = 12</math> and <math>JN = 7</math></p>  <p><math>JM = \underline{12}</math>  <math>JL = \underline{14}</math>  <math>MN = \underline{9.7}</math>  <math>MK = \underline{19.4}</math></p> <p><math>MN: x^2 + 7^2 = 12^2</math>  <math>x^2 = 95</math>  <math>x = 9.7</math></p>	<p>2. <math>EF = 23</math> and <math>DF = 40</math></p>  <p><math>GF = \underline{23}</math>  <math>HF = \underline{20}</math>  <math>GH = \underline{11.4}</math>  <math>GE = \underline{22.8}</math></p> <p><math>GH: x^2 + 20^2 = 23^2</math>  <math>x^2 = 129</math>  <math>x = 11.4</math></p>
<p>3. <math>RT = 22</math> and <math>US = 18</math></p>  <p><math>VT = \underline{11}</math>  <math>UV = \underline{9}</math>  <math>RS = \underline{14.2}</math>  <math>ST = \underline{14.2}</math></p> <p><math>RS: 9^2 + 11^2 = x^2</math>  <math>202 = x^2</math>  <math>14.2 = x</math></p>	<p>4.</p>  <p><math>m\angle 1 = \underline{52^\circ}</math>      <math>m\angle 5 = \underline{38^\circ}</math>  <math>m\angle 2 = \underline{52^\circ}</math>      <math>m\angle 6 = \underline{52^\circ}</math>  <math>m\angle 3 = \underline{38^\circ}</math>      <math>m\angle 7 = \underline{52^\circ}</math>  <math>m\angle 4 = \underline{38^\circ}</math>      <math>m\angle 8 = \underline{90^\circ}</math></p>
<p>5. <math>ZY = 34</math>, <math>WY = 38</math>, and <math>m\angle ZXY = 34^\circ</math>.</p>  <p><math>ZV: x^2 + 19^2 = 34^2</math>  <math>x^2 = 795</math>  <math>x = 28.2</math></p>	<p><math>WZ = \underline{34}</math>      <math>m\angle WXZ = \underline{34^\circ}</math>  <math>WY = \underline{19}</math>      <math>m\angle WVZ = \underline{90^\circ}</math>  <math>ZV = \underline{28.2}</math>      <math>m\angle ZYW = \underline{56^\circ}</math>  <math>ZX = \underline{56.4}</math>      <math>m\angle XYW = \underline{56^\circ}</math></p>

# Properties of SQUARES

A square has ALL the properties of a parallelogram, rectangle, and rhombus!

- Opposite sides are congruent.
- Opposite sides are parallel.
- Opposite angles are congruent.
- Consecutive angles are supplementary.
- Diagonals bisect each other.
- Four right angles.
- Diagonals are congruent.
- Four congruent sides.
- Diagonals are perpendicular.
- Diagonals bisect opposite angles.

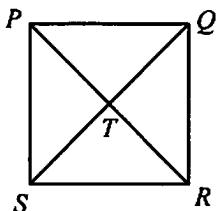
6. If  $ABCD$  is a square and  $AD = 11$ , find each missing value.



$$\begin{aligned} AC &: 11^2 + 11^2 = x^2 \\ 242 &= x^2 \\ x &= 15.6 \end{aligned}$$

$$\begin{aligned} BC &= 11 \\ m\angle DAB &= 90^\circ \\ AC &= 15.6 \\ m\angle AEB &= 90^\circ \\ BD &= 15.6 \\ m\angle CBD &= 45^\circ \\ EC &= 7.8 \\ m\angle BAC &= 45^\circ \end{aligned}$$

7. If  $PQRS$  is a square and  $TR = 17$ , find each missing value.



$$\begin{aligned} PQ &: 17^2 + 17^2 = x^2 \\ 578 &= x^2 \\ x &= 24 \end{aligned}$$

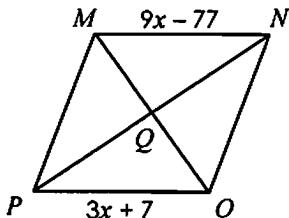
$$\begin{aligned} PR &= 34 \\ m\angle PRS &= 45^\circ \\ QS &= 34 \\ m\angle STR &= 90^\circ \\ QT &= 17 \\ m\angle PSR &= 90^\circ \\ PQ &= 24 \\ m\angle QPR &= 45^\circ \end{aligned}$$

8. If  $MNOP$  is a rhombus, find  $MP$ .

$$\begin{aligned} 9x - 77 &= 3x + 7 \\ 6x &= 84 \\ x &= 14 \end{aligned}$$

$$PO: 3(14) + 7 = 49$$

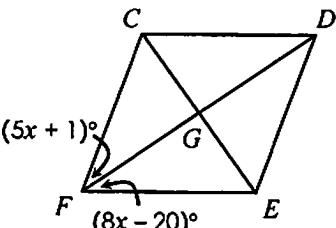
$$\boxed{MP = 49}$$



9. If  $CDEF$  is a rhombus, find  $m\angle FED$ .

$$\begin{aligned} 5x + 1 &= 8x - 20 \\ 3x &= 21 \\ x &= 7 \end{aligned}$$

$$\begin{aligned} m\angle CFG &= 5(1) + 1 \\ &= 36^\circ \\ m\angle CFE &= 72^\circ \end{aligned}$$



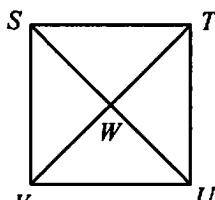
$$\boxed{m\angle FED = 108^\circ}$$

10. If  $STUV$  is a square with  $SW = 2x + 13$  and  $WU = 8x - 41$ , find  $VT$ .

$$\begin{aligned} 8x - 41 &= 2x + 13 \\ 6x &= 54 \\ x &= 9 \end{aligned}$$

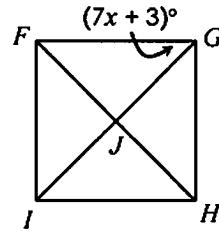
$$SW = 2(9) + 13 = 31$$

$$VT = 2(31) = \boxed{62}$$



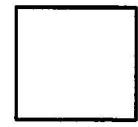
11. If  $FGHI$  is a square, solve for  $x$ .

$$\begin{aligned} 7x + 3 &= 45 \\ 7x &= 42 \\ x &= 6 \end{aligned}$$



Name: \_\_\_\_\_

## Unit 8: Polygons &amp; Quadrilaterals



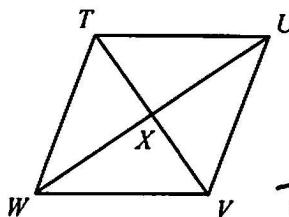
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## Homework 5: Rhombi and Squares

\*\* This is a 2-page document! \*\*

Directions: If each quadrilateral below is a rhombus, find the missing measures.

1.  $UV = 8$  and  $WX = 5$



$TU = 8$

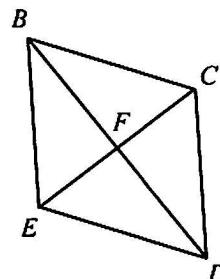
$WU = 10$

$TX = 6.2$

$TV = 12.4$

$$\begin{aligned} TX: \quad & X^2 + 5^2 = 8^2 \\ & X^2 = 39 \\ & X = 6.2 \end{aligned}$$

2.  $BC = 28$  and  $BD = 32$



$CD = 28$

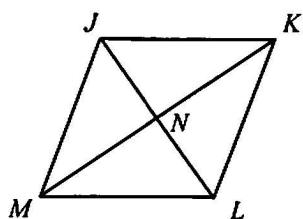
$FD = 16$

$EF = 23$

$EC = 46$

$$\begin{aligned} EF: \quad & X^2 + 16^2 = 28^2 \\ & X^2 = 528 \\ & X = 23 \end{aligned}$$

3.  $MK = 24$ ,  $JL = 20$ , and  $m\angle MJL = 50^\circ$



$$\begin{aligned} ML: \quad & 10^2 + 12^2 = X^2 \\ & 244 = X^2 \\ & X = 15.6 \end{aligned}$$

$NK = 12$

$NL = 10$

$ML = 15.6$

$JM = 15.6$

$m\angle KNL = 90^\circ$

$m\angle KJL = 50^\circ$

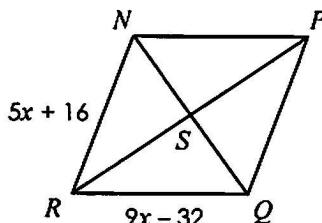
$m\angle MLK = 100^\circ$

$m\angle JKM = 40^\circ$

$m\angle JML = 80^\circ$

4. Find  $PQ$ .

$$\begin{aligned} 9x - 32 &= 5x + 16 \\ 4x &= 48 \\ x &= 12 \end{aligned}$$

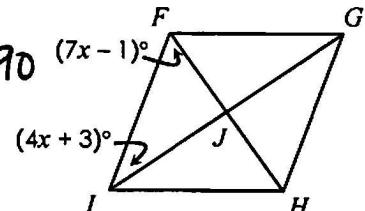


$NR: 5(12) + 16 = 76$

$PQ = 76$

5. Find  $m\angle HGI$ .

$$\begin{aligned} 7x - 1 + 4x + 3 &= 90 \\ 11x &= 88 \\ x &= 8 \end{aligned}$$

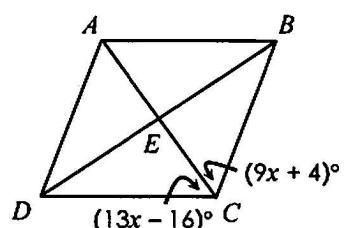


$m\angle FIJ: 4(8) + 3 = 35^\circ$

$m\angle HGI = 35^\circ$

6. Find  $m\angle ADB$ .

$$\begin{aligned} 13x - 16 &= 9x + 4 \\ 4x &= 20 \\ x &= 5 \end{aligned}$$



$m\angle BCE: 9(5) + 4 = 49^\circ$

$m\angle ADB = 41^\circ$

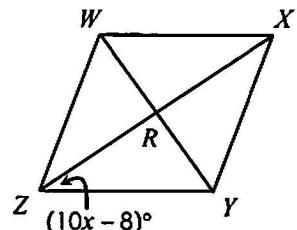
7. If  $m\angle XYZ = 136^\circ$ , solve for  $x$ .

$m\angle YZW = 44^\circ$

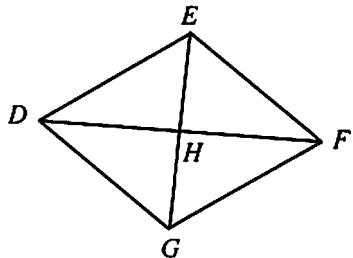
$10x - 8 = 22$

$10x = 30$

$x = 3$



8. If  $DE = 16x - 3$ ,  $EF = 9x + 11$ , and  $DF = 52$ , find  $HG$ .



$$16x - 3 = 9x + 11$$

$$7x = 14$$

$$x = 2$$

$$DE: 16(2) - 3 = 29$$

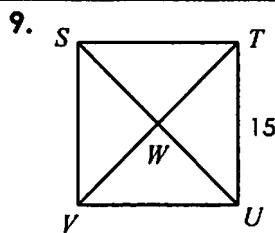
$$x^2 + 26^2 = 29^2$$

$$x^2 = 165$$

$$x = 12.8$$

$$HG = 12.8$$

**Directions:** If each quadrilateral below is a square, find the missing measures.



$$VU = \underline{15}$$

$$SU = \underline{21.2}$$

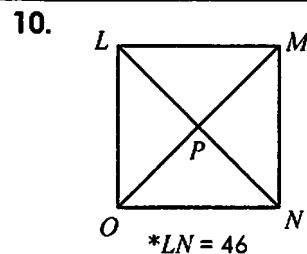
$$TV = \underline{21.2}$$

$$SW = \underline{10.6}$$

$$15^2 + 15^2 = x^2$$

$$450 = x^2$$

$$x = 21.2$$



$$OM = \underline{46}$$

$$PN = \underline{23}$$

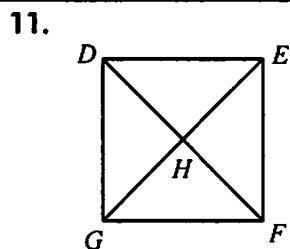
$$ON = \underline{32.5}$$

$$MN = \underline{32.5}$$

$$23^2 + 23^2 = x^2$$

$$1058 = x^2$$

$$x = 32.5$$



$$m\angle EFG = \underline{90^\circ}$$

$$m\angle GDH = \underline{45^\circ}$$

$$m\angle FEG = \underline{45^\circ}$$

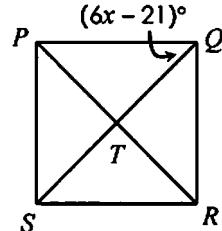
$$m\angle DHG = \underline{90^\circ}$$

12. Solve for  $x$ .

$$6x - 21 = 45$$

$$6x = 66$$

$$x = 11$$



13. Which quadrilaterals always have diagonals that are congruent?

- Parallelograms
- Rectangles
- Rhombi
- Squares

14. Which quadrilaterals always have consecutive angles that are supplementary?

- Parallelograms
- Rectangles
- Rhombi
- Squares

15. Which quadrilaterals always have diagonals that are perpendicular?

- Parallelograms
- Rectangles
- Rhombi
- Squares

16. Which quadrilaterals always have diagonals that bisect each other?

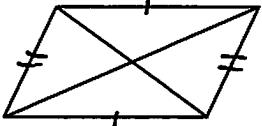
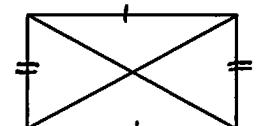
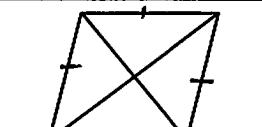
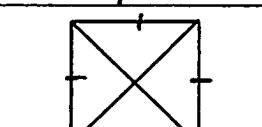
- Parallelograms
- Rectangles
- Rhombi
- Squares

# COORDINATE GEOMETRY: Name that Quadrilateral!

To classify a quadrilateral as a parallelogram, rectangle, rhombus, or square:

> Step 1: Check congruency of sides.

> Step 2: Check congruency of diagonals.

<b>CASE 1</b> (Parallelogram)	Opposite sides are congruent and diagonals are NOT congruent.	
<b>CASE 2</b> (Rectangle)	Opposite sides are congruent and diagonals are congruent.	
<b>CASE 3</b> (Rhombus)	All four sides are congruent and diagonals are NOT congruent.	
<b>CASE 4</b> (Square)	All four sides are congruent and diagonals are congruent.	

Directions: Given the vertices, determine the quadrilaterals most specific classification.

1)  $A(9, -4), B(8, -2), C(2, -5), D(3, -7)$

$$\begin{aligned} AB &= \sqrt{(8-9)^2 + (-2+4)^2} = \sqrt{1+4} = \sqrt{5} \\ BC &= \sqrt{(2-8)^2 + (-5+2)^2} = \sqrt{36+9} = \sqrt{45} = 3\sqrt{5} \\ CD &= \sqrt{(3-2)^2 + (-7+5)^2} = \sqrt{1+4} = \sqrt{5} \\ AD &= \sqrt{(3-9)^2 + (-7+4)^2} = \sqrt{36+9} = \sqrt{45} = 3\sqrt{5} \end{aligned}$$

Parallelogram  
or  
Rectangle

$$\begin{aligned} AC &= \sqrt{(2-9)^2 + (-5+4)^2} = \sqrt{49+1} = \sqrt{50} = 5\sqrt{2} \\ BD &= \sqrt{(3-8)^2 + (-7+2)^2} = \sqrt{25+25} = \sqrt{50} = 5\sqrt{2} \end{aligned}$$

$ABCD$  is a Rectangle.

- ②  $Q(-2, -7)$ ,  $R(1, -5)$ ,  $S(4, -7)$ ,  $T(1, -9)$

$$QR = \sqrt{(1+2)^2 + (-5+7)^2} = \sqrt{9+4} = \sqrt{13}$$

$$RS = \sqrt{(4-1)^2 + (-7+5)^2} = \sqrt{9+4} = \sqrt{13}$$

$$ST = \sqrt{(1-4)^2 + (-9+7)^2} = \sqrt{9+4} = \sqrt{13}$$

$$QT = \sqrt{(1+2)^2 + (-9+7)^2} = \sqrt{9+4} = \sqrt{13}$$

Rhombus  
or  
Square

$$QS = \sqrt{(4+2)^2 + (-7+7)^2} = \sqrt{36} = 6$$

$$RT = \sqrt{(1-1)^2 + (-9+5)^2} = \sqrt{16} = 4$$

$QRST$  is a Rhombus.

- ③  $J(5, -1)$ ,  $K(8, 2)$ ,  $L(11, 10)$ ,  $M(8, 7)$

$$JK = \sqrt{(8-5)^2 + (2+1)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

$$KL = \sqrt{(11-8)^2 + (10-2)^2} = \sqrt{9+64} = \sqrt{73}$$

$$LM = \sqrt{(8-11)^2 + (7-10)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

$$JM = \sqrt{(8-5)^2 + (7+1)^2} = \sqrt{9+64} = \sqrt{73}$$

Parallelogram  
or  
Rectangle

$$JL = \sqrt{(11-5)^2 + (10+1)^2} = \sqrt{36+121} = \sqrt{157}$$

$$KM = \sqrt{(8-8)^2 + (7-2)^2} = \sqrt{25} = 5$$

$JKLM$  is a Parallelogram.

- ④  $W(-4, -3), X(1, -2), Y(2, -7), Z(-3, -8)$

$$WX = \sqrt{(1+4)^2 + (-2+3)^2} = \sqrt{25+1} = \sqrt{26}$$

$$XY = \sqrt{(2-1)^2 + (-7+2)^2} = \sqrt{1+25} = \sqrt{26}$$

$$YZ = \sqrt{(-3-2)^2 + (-8+7)^2} = \sqrt{25+1} = \sqrt{26}$$

$$WZ = \sqrt{(-3+4)^2 + (-8+3)^2} = \sqrt{1+25} = \sqrt{26}$$

Square  
or  
Rhombus

$$WY = \sqrt{(2+4)^2 + (-7+3)^2} = \sqrt{36+16} = \sqrt{52} = 2\sqrt{13}$$

$$XZ = \sqrt{(-3-1)^2 + (-8+2)^2} = \sqrt{16+36} = \sqrt{52} = 2\sqrt{13}$$

$WXYZ$  is a Square.

- ⑤  $D(-5, 9), E(-3, 6), F(-6, -2), G(-8, 1)$

$$DE = \sqrt{(-3+5)^2 + (6-9)^2} = \sqrt{4+9} = \sqrt{13}$$

$$EF = \sqrt{(-6+3)^2 + (-2-6)^2} = \sqrt{9+64} = \sqrt{73}$$

$$FG = \sqrt{(-8+6)^2 + (1+2)^2} = \sqrt{4+9} = \sqrt{13}$$

$$DG = \sqrt{(-8+5)^2 + (1-9)^2} = \sqrt{9+64} = \sqrt{73}$$

Parallelogram  
or  
Rectangle

$$DF = \sqrt{(-6+5)^2 + (-2-9)^2} = \sqrt{1+121} = \sqrt{122}$$

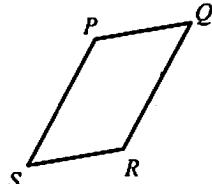
$$EG = \sqrt{(-8+3)^2 + (1-6)^2} = \sqrt{25+25} = \sqrt{50} = 5\sqrt{2}$$

$DEFG$  is a Parallelogram.

# QUADRILATERALS in the COORDINATE PLANE

**Directions:** Use your knowledge of slope, distance, midpoint, and the properties of quadrilaterals to answer the following questions.

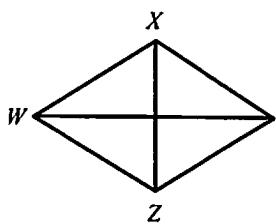
1. On parallelogram  $PQRS$  below, if  $P$  is located at  $(-1, 6)$  and  $S$  is located at  $(-7, -3)$ , what is the slope of  $\overline{QR}$ ?



$$m \overline{PS} = \frac{-3-6}{-7+1} = \frac{-9}{-6} = \frac{3}{2}$$

$$m \overline{QR} = \frac{3}{2}$$

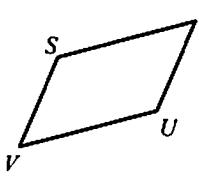
3. On rhombus  $WXYZ$ , if  $W$  is located at  $(-5, -2)$  and  $Y$  is located at  $(3, -2)$ , what is the slope of  $\overline{XZ}$ ?



$$m \overline{WY} = \frac{-2+2}{3+5} = \frac{0}{8} = 0$$

$$m \overline{XZ} = \text{undefined}$$

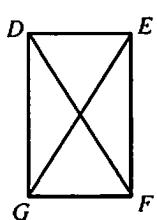
5. On parallelogram  $STUV$  below, if  $S$  is located at  $(-4, 1)$  and  $T$  is located at  $(5, 3)$ , what is the length of  $\overline{VU}$ ?



$$\begin{aligned} ST &= \sqrt{(-4-5)^2 + (1-3)^2} \\ &= \sqrt{81+4} \\ &= \sqrt{85} \end{aligned}$$

$$VU = \sqrt{85}$$

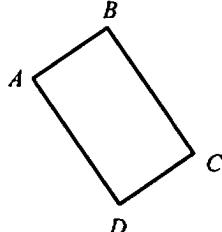
7. On rectangle  $DEFG$  below, if  $D$  is located at  $(-1, -1)$  and  $F$  is located at  $(4, -8)$ , what is the length of  $\overline{GE}$ ?



$$\begin{aligned} DF &= \sqrt{(-1-4)^2 + (-1+8)^2} \\ &= \sqrt{25+49} \\ &= \sqrt{74} \end{aligned}$$

$$GE = \sqrt{74}$$

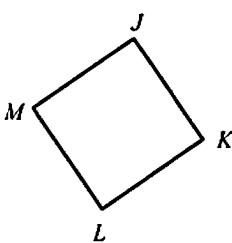
2. On rectangle  $ABCD$  below, if  $A$  is located at  $(3, 4)$  and  $B$  is located at  $(7, 6)$ , is the slope of  $\overline{BC}$ ?



$$m \overline{AB} = \frac{6-4}{7-3} = \frac{2}{4} = \frac{1}{2}$$

$$m \overline{BC} = -2$$

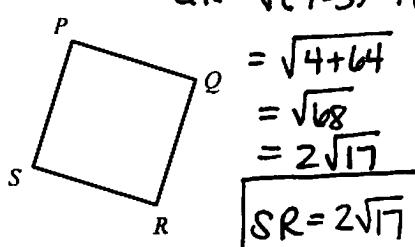
4. On square  $JKLM$  below, if  $J$  is located at  $(-2, 5)$  and  $K$  is located at  $(2, 2)$ , what is the slope of  $\overline{LK}$ ?



$$m \overline{JK} = \frac{2-5}{2+2} = -\frac{3}{4}$$

$$m \overline{LK} = \frac{4}{3}$$

6. On square  $PQRS$  below, if  $Q$  is located at  $(7, 0)$  and  $R$  is located at  $(5, -8)$ , what is the length of  $\overline{SR}$ ?

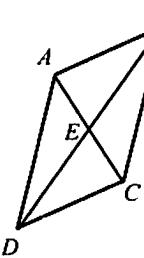


$$QR = \sqrt{(7-5)^2 + (0+8)^2}$$

$$\begin{aligned} &= \sqrt{4+64} \\ &= \sqrt{68} \\ &= 2\sqrt{17} \end{aligned}$$

$$SR = 2\sqrt{17}$$

8. On parallelogram  $ABCD$  below, if  $A(1, 1)$ ,  $B(8, 5)$ ,  $C(5, -5)$  and  $D(-2, -9)$ , what are the coordinates of point  $E$ ?



$$E = \left( \frac{-2+8}{2}, \frac{-9+5}{2} \right)$$

$$= (3, -2)$$

Name: \_\_\_\_\_

**Unit 8: Polygons & Quadrilaterals**

Date: \_\_\_\_\_ Per: \_\_\_\_\_

**Homework 6: Classifying Quadrilaterals  
in the Coordinate Plane****\*\* This is a 2-page document! \*\***

**Directions:** Given the vertices, determine the quadrilaterals most specific classification:  
Parallelogram, Rectangle, Rhombus, or Square. Justify your answer using the distance formula.

1.  $S(-9, 14), T(1, 10), U(-3, 0), V(-13, 4)$

$$\begin{aligned} ST &= \sqrt{(1+9)^2 + (10-14)^2} = \sqrt{100+16} = \sqrt{116} = 2\sqrt{29} \\ TU &= \sqrt{(-3-1)^2 + (0-10)^2} = \sqrt{16+100} = \sqrt{116} = 2\sqrt{29} \\ UV &= \sqrt{(-13+3)^2 + (4-0)^2} = \sqrt{100+16} = \sqrt{116} = 2\sqrt{29} \\ SV &= \sqrt{(-13+9)^2 + (4-14)^2} = \sqrt{16+100} = \sqrt{116} = 2\sqrt{29} \end{aligned} \quad \left. \begin{array}{l} \text{Rhombus} \\ \text{or} \\ \text{Square} \end{array} \right\}$$

$$SU = \sqrt{(-3+9)^2 + (0-14)^2} = \sqrt{36+196} = \sqrt{232} = 2\sqrt{58}$$

$$TV = \sqrt{(-13-1)^2 + (4-10)^2} = \sqrt{196+36} = \sqrt{232} = 2\sqrt{58}$$

$STUV$  is a Square.

2.  $E(-7, -4), F(2, -3), G(0, -7), H(-9, -8)$

$$\begin{aligned} EF &= \sqrt{(2+7)^2 + (-3+4)^2} = \sqrt{81+1} = \sqrt{82} \\ FG &= \sqrt{(0-2)^2 + (-7+3)^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5} \\ GH &= \sqrt{(-9-0)^2 + (-8+7)^2} = \sqrt{81+1} = \sqrt{82} \\ EH &= \sqrt{(-9+7)^2 + (-8+4)^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5} \end{aligned} \quad \left. \begin{array}{l} \text{Parallelogram} \\ \text{or} \\ \text{Rectangle} \end{array} \right\}$$

$$EG = \sqrt{(0+7)^2 + (-7+4)^2} = \sqrt{49+9} = \sqrt{58}$$

$$FH = \sqrt{(-9-2)^2 + (-8+3)^2} = \sqrt{121+25} = \sqrt{146}$$

$EFGH$  is a Parallelogram.

3.  $A(-5, 8), B(-2, 14), C(12, 7), D(9, 1)$

$$AB = \sqrt{(-2+5)^2 + (14-8)^2} = \sqrt{9+36} = \sqrt{45} = 3\sqrt{5}$$

$$BC = \sqrt{(12+2)^2 + (7-14)^2} = \sqrt{196+49} = \sqrt{245} = 7\sqrt{5}$$

$$CD = \sqrt{(9-12)^2 + (1-7)^2} = \sqrt{9+36} = \sqrt{45} = 3\sqrt{5}$$

$$AD = \sqrt{(9+5)^2 + (1-8)^2} = \sqrt{196+49} = \sqrt{245} = 7\sqrt{5}$$

Rectangle  
or  
Parallelogram

$$AC = \sqrt{(12+5)^2 + (7-8)^2} = \sqrt{289+1} = \sqrt{290}$$

$$BD = \sqrt{(9+2)^2 + (1-14)^2} = \sqrt{121+169} = \sqrt{290}$$

$ABCD$  is a Rectangle.

4.  $K(5, -3), L(7, 1), M(9, -3), N(7, -7)$

$$KL = \sqrt{(7-5)^2 + (1+3)^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$$

$$LM = \sqrt{(9-7)^2 + (-3-1)^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$$

$$MN = \sqrt{(7-9)^2 + (-7+3)^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$$

$$KN = \sqrt{(7-5)^2 + (-7+3)^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$$

Rhombus  
or  
Square

$$KM = \sqrt{(9-5)^2 + (-3+3)^2} = \sqrt{16} = 4$$

$$LN = \sqrt{(7-7)^2 + (-7-1)^2} = \sqrt{64} = 8$$

$KLMN$  is a Rhombus.

Name: \_\_\_\_\_

**Geometry**

Date: \_\_\_\_\_ Per: \_\_\_\_\_

**Unit 8: Polygons & Quadrilaterals****Quiz 8-2: Parallelograms, Rectangles, Rhombi, & Squares**

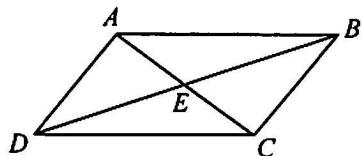
1. Which quadrilaterals always have opposite angles that are congruent?

- Parallelograms
- Rectangles
- Rhombi
- Squares

2. Which quadrilaterals always have diagonals that bisect opposite angles?

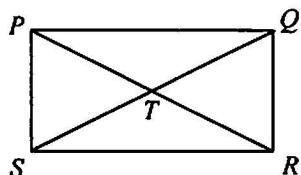
- Parallelograms
- Rectangles
- Rhombi
- Squares

3. If  $ABCD$  is a parallelogram,  $AD = 14$ ,  $EC = 11$ ,  $m\angle ABC = 64^\circ$ ,  $m\angle DAC = 71^\circ$ , and  $m\angle BDC = 25^\circ$ , find each measure.



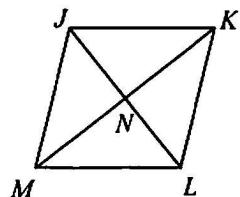
- a)  $BC = \underline{14}$       d)  $m\angle ABD = \underline{25^\circ}$   
 b)  $AC = \underline{22}$       e)  $m\angle ACD = \underline{45^\circ}$   
 c)  $m\angle DAB = \underline{116^\circ}$       f)  $m\angle ADB = \underline{39^\circ}$

4. If  $PQRS$  is a rectangle,  $ST = 12$ , and  $m\angle PRS = 23^\circ$ , find each measure.



- a)  $SQ = \underline{24}$       d)  $m\angle PSR = \underline{90^\circ}$   
 b)  $PR = \underline{24}$       e)  $m\angle SQR = \underline{67^\circ}$   
 c)  $m\angle QPR = \underline{23^\circ}$       f)  $m\angle PTQ = \underline{134^\circ}$

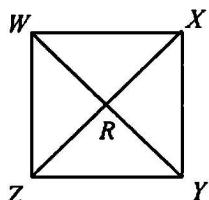
5. If  $JKLM$  is a rhombus,  $MK = 30$ ,  $NL = 13$ , and  $m\angle MKL = 41^\circ$ , find each measure.



$$\begin{aligned} KL: 13^2 + 15^2 &= x^2 \\ 394 &= x^2 \\ x &= 19.8 \end{aligned}$$

- a)  $NK = \underline{15}$       e)  $m\angle JML = \underline{82^\circ}$   
 b)  $JL = \underline{26}$       f)  $m\angle MLK = \underline{98^\circ}$   
 c)  $KL = \underline{19.8}$       g)  $m\angle MNL = \underline{90^\circ}$   
 d)  $m\angle JKM = \underline{41^\circ}$       h)  $m\angle KJL = \underline{49^\circ}$

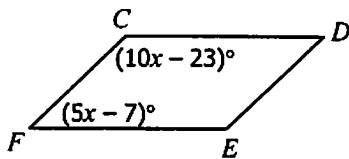
6. If  $WXYZ$  is a square with  $WZ = 27$ , find each measure.



$$\begin{aligned} 27^2 + 27^2 &= x^2 \\ 1458 &= x^2 \\ x &= 38.2 \end{aligned}$$

- a)  $ZY = \underline{27}$       d)  $m\angle WRZ = \underline{90^\circ}$   
 b)  $WY = \underline{38.2}$       e)  $m\angle XYZ = \underline{90^\circ}$   
 c)  $RX = \underline{19.1}$       f)  $m\angle ZWY = \underline{45^\circ}$

7. If  $CDEF$  is a parallelogram, find  $m\angle FCD$ .

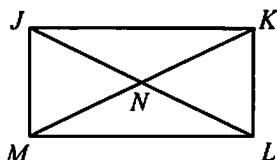


$$\begin{aligned}10x - 23 + 5x - 7 &= 180 \\15x - 30 &= 180 \\15x &= 210 \\x &= 14\end{aligned}$$

7.  $117^\circ$

$$\begin{aligned}m\angle FCD &= 10(14) - 23 \\&= 117^\circ\end{aligned}$$

8. If  $JKLM$  is a rectangle,  $JL = 2x + 5$ , and  $MK = 7x - 40$ , find  $MK$ .

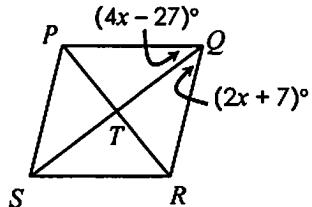


$$\begin{aligned}7x - 40 &= 2x + 5 \\5x &= 45 \\x &= 9\end{aligned}$$

$$\begin{aligned}MK &= 7(9) - 40 \\&= 23\end{aligned}$$

8.  $23$

9. If  $PQRS$  is a rhombus, find  $m\angle PQR$ .



$$\begin{aligned}4x - 27 &= 2x + 7 \\2x &= 34 \\x &= 17\end{aligned}$$

$$m\angle RQS = 2(17) + 7 = 41^\circ$$

$$m\angle PQR = 2(41) = 82^\circ$$

10. Quadrilateral  $BCDE$  has vertices  $B(-1, -1)$ ,  $C(6, -2)$ ,  $D(5, -9)$ , and  $E(-2, -8)$ . Determine the most precise classification of  $BCDE$ : a parallelogram, rectangle, rhombus, or square. Use the distance formula to justify your answer.

$$\begin{aligned}BC &= \sqrt{(6+1)^2 + (-2+1)^2} = \sqrt{49+1} = \sqrt{50} = 5\sqrt{2} \\CD &= \sqrt{(5-6)^2 + (-9+2)^2} = \sqrt{1+49} = \sqrt{50} = 5\sqrt{2} \\DE &= \sqrt{(-2-5)^2 + (-8+9)^2} = \sqrt{49+1} = \sqrt{50} = 5\sqrt{2} \\BE &= \sqrt{(-2+1)^2 + (-8+1)^2} = \sqrt{1+49} = \sqrt{50} = 5\sqrt{2}\end{aligned}$$

Rhombus  
or  
Square

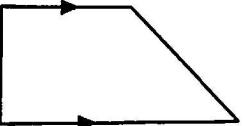
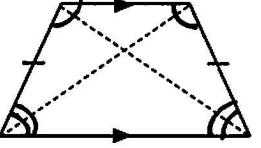
$$BD = \sqrt{(5+1)^2 + (-9+1)^2} = \sqrt{36+64} = \sqrt{100} = 10$$

$$CE = \sqrt{(-2-6)^2 + (-8+2)^2} = \sqrt{64+36} = \sqrt{100} = 10$$

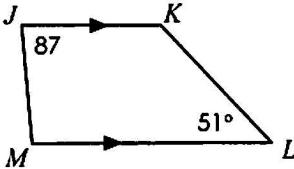
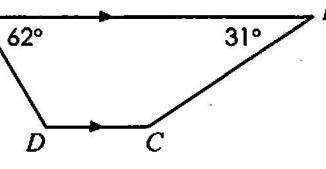
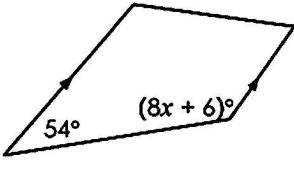
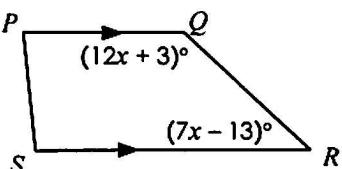
10.  $BCDE$  is a Square

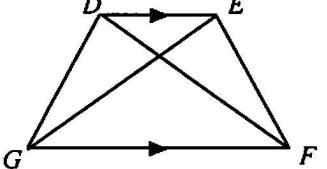
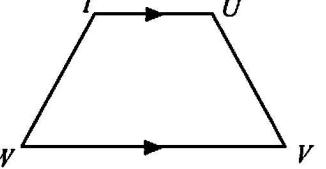
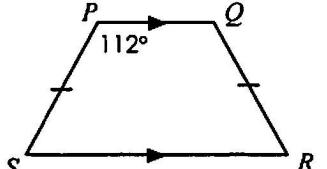
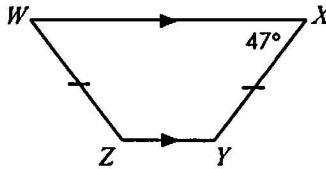
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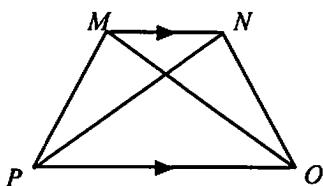
Main Ideas/Questions	Notes/Examples
<b>NON-ISOSCELES Trapezoids</b>	 <p><b>Properties of Non-Isosceles Trapezoids:</b></p> <ul style="list-style-type: none"> <li>Only ONE pair of opposite sides parallel.</li> <li>Consecutive angles between parallel lines are supplementary.</li> </ul>
<b>ISOSCELES Trapezoids</b>	 <p>Isosceles trapezoids have the same properties as non-isosceles trapezoids, plus these:</p> <ul style="list-style-type: none"> <li>Non-parallel sides (legs) are congruent.</li> <li>Diagonals are congruent.</li> <li>Base angles are congruent.</li> <li>Opposite angles are supplementary.</li> </ul>

**Directions:** Find each missing value on the trapezoids below.

<p>1.</p>  $m\angle K = \underline{129^\circ}$ $m\angle M = \underline{93^\circ}$	<p>2.</p>  $m\angle C = \underline{149^\circ}$ $m\angle D = \underline{118^\circ}$
<p>3. Solve for <math>x</math>.</p> $8x + 60 = 180$ $8x = 120$ $\boxed{x = 15}$ 	<p>4. Find <math>m\angle R</math>.</p> $19x - 10 = 180$ $19x = 190$ $x = 10$ $m\angle R = 7(10) - 13$ $\boxed{= 57^\circ}$ 

<p>5. <math>DEFG</math> is an isosceles trapezoid.</p>  $\overline{DG} \cong \underline{\overline{EF}}$ $\overline{DF} \cong \underline{\overline{GE}}$	<p>6. <math>TUVW</math> is an isosceles trapezoid.</p>  $\angle T \cong \underline{\angle U}$ $\angle V \cong \underline{\angle W}$
<p>7.</p>  $m\angle Q = \underline{112^\circ}$ $m\angle R = \underline{68^\circ}$ $m\angle S = \underline{68^\circ}$	<p>8.</p>  $m\angle W = \underline{47^\circ}$ $m\angle Y = \underline{133^\circ}$ $m\angle Z = \underline{133^\circ}$

9. If  $MNOP$  is an isosceles trapezoid,  $MP = 16x - 13$ ,  $NO = 9x + 8$ ,  $PN = 5y + 19$ , and  $MO = 12y - 37$ , solve for  $x$  and  $y$ .



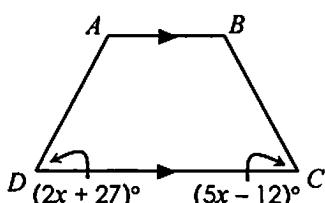
$$16x - 13 = 9x + 8$$

$$\begin{aligned} 7x &= 21 \\ x &= 3 \end{aligned}$$

$$5y + 19 = 12y - 37$$

$$\begin{aligned} 56 &= 7y \\ y &= 8 \end{aligned}$$

10. If  $ABCD$  is an isosceles trapezoid, find each missing angle.



$$5x - 12 = 2x + 27$$

$$\begin{aligned} 3x &= 39 \\ x &= 13 \end{aligned}$$

$$m\angle D = 2(13) + 27 = 53^\circ$$

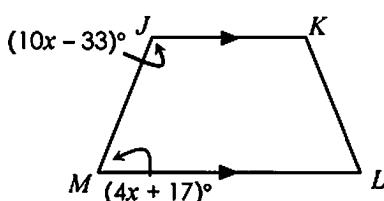
$$m\angle A = 127^\circ$$

$$m\angle B = 127^\circ$$

$$m\angle C = 53^\circ$$

$$m\angle D = 53^\circ$$

11. If  $JKLM$  is an isosceles trapezoid, find each missing angle.



$$14x - 16 = 180$$

$$14x = 196$$

$$x = 14$$

$$m\angle M = 4(14) + 17 = 73^\circ$$

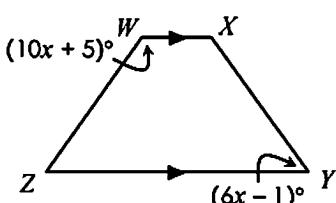
$$m\angle J = 107^\circ$$

$$m\angle K = 107^\circ$$

$$m\angle L = 73^\circ$$

$$m\angle M = 73^\circ$$

12. If  $WXYZ$  is an isosceles trapezoid, find each missing angle.



$$16x + 4 = 180$$

$$16x = 176$$

$$x = 11$$

$$m\angle Y = 6(11) - 1 = 65^\circ$$

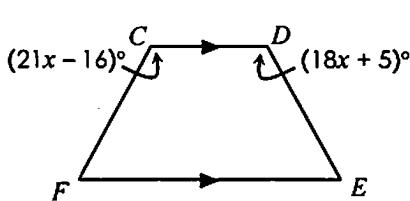
$$m\angle W = 115^\circ$$

$$m\angle X = 115^\circ$$

$$m\angle Y = 65^\circ$$

$$m\angle Z = 65^\circ$$

13. If  $CDEF$  is an isosceles trapezoid, find each missing angle.



$$21x - 16 = 18x + 5$$

$$3x = 21$$

$$x = 7$$

$$m\angle D = 18(7) + 5 = 131^\circ$$

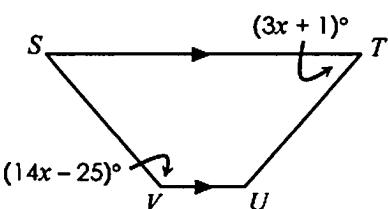
$$m\angle C = 131^\circ$$

$$m\angle D = 131^\circ$$

$$m\angle E = 49^\circ$$

$$m\angle F = 49^\circ$$

14. If  $STUV$  is an isosceles trapezoid, find each missing angle.



$$17x - 24 = 180$$

$$17x = 204$$

$$x = 12$$

$$m\angle T = 3(12) + 1 = 37^\circ$$

$$m\angle S = 37^\circ$$

$$m\angle T = 37^\circ$$

$$m\angle U = 143^\circ$$

$$m\angle V = 143^\circ$$

Name:

Date:

Topic:

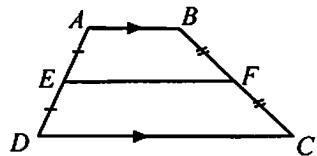
Class:

## Main Ideas/Questions

**MIDSEGMENT  
of a TRAPEZOID**

## Notes/Examples

The midsegment of a trapezoid connects the midpoints of the legs:



If  $\overline{EF}$  is the midsegment of trapezoid  $ABCD$ , then:

- $\overline{AB} \parallel \overline{CD} \parallel \overline{EF}$
- $EF = \frac{AB+CD}{2}$

Directions: Use the trapezoid above for questions 1-4.

1. If  $AB = 14$  and  $DC = 26$ , find  $EF$ .

$$EF = \frac{14+26}{2} = \frac{40}{2} = 20$$

2. If  $AB = 7$  and  $DC = 31$ , find  $EF$ .

$$EF = \frac{7+31}{2} = \frac{38}{2} = 19$$

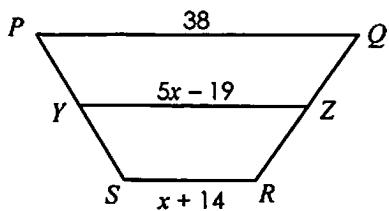
3. If  $EF = 22$  and  $DC = 38$ , find  $AB$ .

$$22 = \frac{AB+38}{2} \quad 44 = AB+38 \quad 6 = AB$$

4. If  $AB = 41$  and  $EF = 47$ , find  $DC$ .

$$47 = \frac{41+CD}{2} \quad 94 = 41+CD \quad 53 = CD$$

5. For trapezoid  $PQRS$ ,  $Y$  and  $Z$  are midpoints of the legs. Find  $YZ$ .



$$\frac{38+x+14}{2} = 5x - 19$$

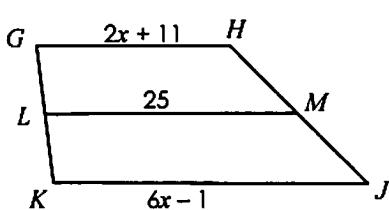
$$x+52=10x-38$$

$$90=9x$$

$$10=x$$

$$YZ = 5(10) - 19 \\ = 31$$

6. For trapezoid  $GHJK$ ,  $L$  and  $M$  are midpoints of the legs. Find  $KJ$ .



$$\frac{2x+11+6x-1}{2} = 25$$

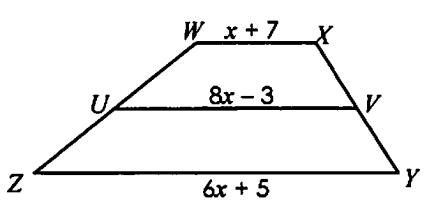
$$8x+10=50$$

$$8x=40$$

$$x=5$$

$$KJ = 6(5) - 1 \\ = 29$$

7. For trapezoid  $WXYZ$ ,  $U$  and  $V$  are midpoints of the legs. Find  $UV$ .



$$\frac{x+7+6x+5}{2} = 8x - 3$$

$$7x+12=16x-6$$

$$18=9x$$

$$2=x$$

$$UV = 8(2) - 3 \\ = 13$$

Name: \_\_\_\_\_

## Unit 8: Polygons &amp; Quadrilaterals



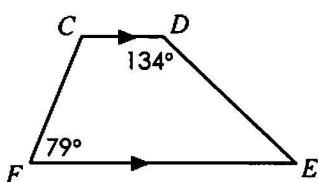
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## Homework 7: Trapezoids

\*\* This is a 2-page document! \*\*

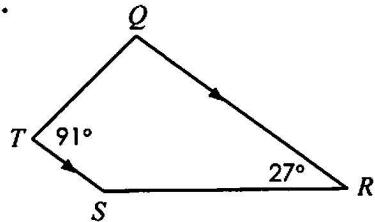
Directions: If each quadrilateral below is a trapezoid, find the missing measures.

1.



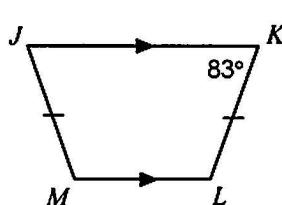
$$\begin{aligned}m\angle C &= 101^\circ \\ m\angle E &= 46^\circ\end{aligned}$$

2.



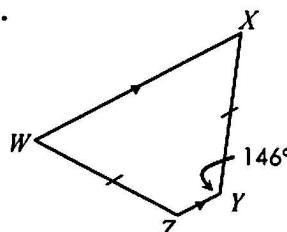
$$\begin{aligned}m\angle Q &= 89^\circ \\ m\angle S &= 153^\circ\end{aligned}$$

3.



$$\begin{aligned}m\angle J &= 83^\circ \\ m\angle L &= 97^\circ \\ m\angle M &= 97^\circ\end{aligned}$$

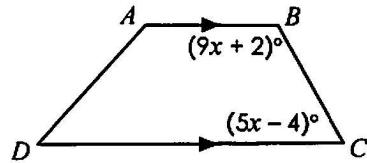
4.



$$\begin{aligned}m\angle W &= 34^\circ \\ m\angle X &= 34^\circ \\ m\angle Z &= 146^\circ\end{aligned}$$

5. Solve for  $x$ .

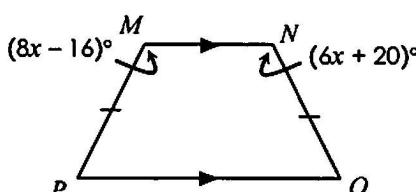
$$\begin{aligned}(14x - 15)^\circ + 139^\circ &= 180^\circ \\ 14x + 124 &= 180 \\ 14x &= 56 \\ x &= 4\end{aligned}$$

6. Find  $m\angle B$ .

$$\begin{aligned}14x - 2 &= 180 \\ 14x &= 182 \\ x &= 13\end{aligned}$$

$$m\angle B = 9(13) + 2 = 119^\circ$$

7.



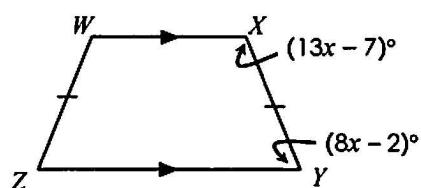
$$8x - 16 = 6x + 20$$

$$\begin{aligned}2x &= 36 \\ x &= 18\end{aligned}$$

$$\begin{aligned}m\angle M &= 8(18) - 16 \\ &= 128^\circ\end{aligned}$$

$$\begin{aligned}m\angle M &= 128^\circ \\ m\angle N &= 128^\circ \\ m\angle O &= 52^\circ \\ m\angle P &= 52^\circ\end{aligned}$$

8.

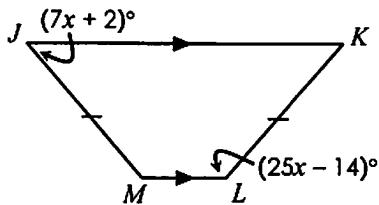


$$\begin{aligned}21x - 9 &= 180 \\ 21x &= 189 \\ x &= 9\end{aligned}$$

$$m\angle Y = 8(9) - 2 = 70^\circ$$

$$\begin{aligned}m\angle W &= 110^\circ \\ m\angle X &= 110^\circ \\ m\angle Y &= 70^\circ \\ m\angle Z &= 70^\circ\end{aligned}$$

9.



$$32x - 12 = 180$$

$$32x = 192$$

$$x = 6$$

$$m\angle J = 44^\circ$$

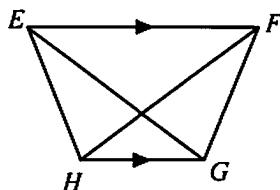
$$m\angle K = 44^\circ$$

$$m\angle L = 136^\circ$$

$$m\angle M = 136^\circ$$

$$m\angle J = 7(6) + 2 = 44^\circ$$

10. If  $EFGH$  is an isosceles trapezoid,  $EH = 4x - 27$ ,  $FG = x + 9$ ,  $EG = 3y + 19$ , and  $FH = 11y - 21$ , solve for  $x$  and  $y$ .



$$4x - 27 = x + 9$$

$$3x = 36$$

$$\boxed{x = 12}$$

$$3y + 19 = 11y - 21$$

$$40 = 8y$$

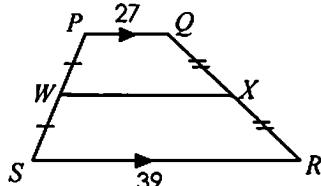
$$\boxed{5 = y}$$

11. Find  $WX$ .

$$\frac{27+39}{2} = WX$$

$$\frac{66}{2} = WX$$

$$\boxed{WX = 33}$$

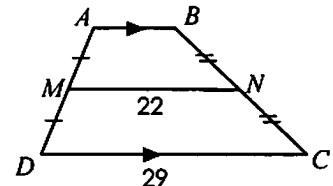


12. Find  $AB$ .

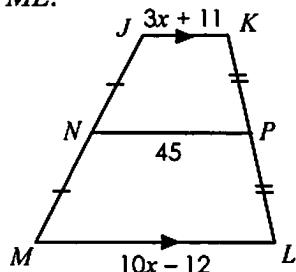
$$\frac{AB + 29}{2} = 22$$

$$AB + 29 = 44$$

$$\boxed{AB = 15}$$



13. Find  $ML$ .



$$\frac{3x + 11 + 10x - 12}{2} = 45$$

$$13x - 1 = 90$$

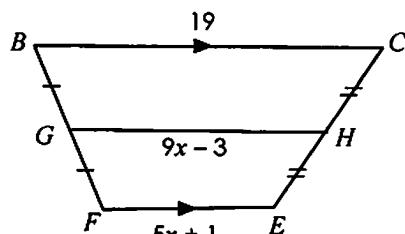
$$13x = 91$$

$$x = 7$$

$$ML = 10(7) - 12$$

$$\boxed{= 58}$$

14. Find  $GH$ .



$$\frac{19 + 5x + 1}{2} = 9x - 3$$

$$5x + 20 = 18x - 6$$

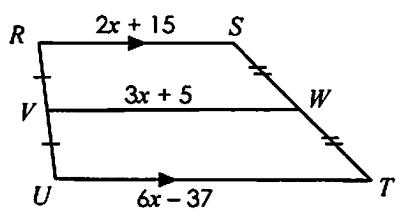
$$26 = 13x$$

$$x = 2$$

$$GH = 9(2) - 3$$

$$\boxed{= 15}$$

15. Find  $RS$ .



$$\frac{2x + 15 + 6x - 37}{2} = 3x + 5$$

$$8x - 22 = 6x + 10$$

$$2x = 32$$

$$x = 16$$

$$RS = 2(16) + 15$$

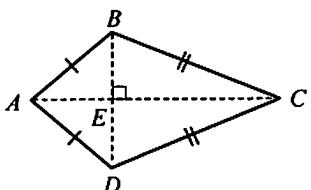
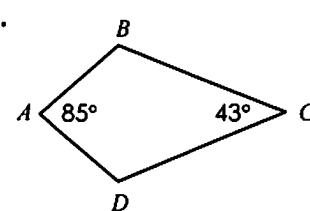
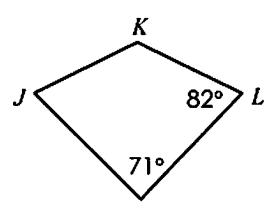
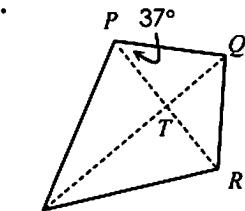
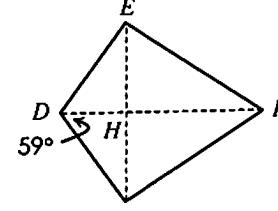
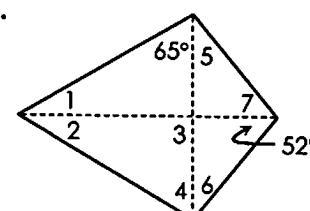
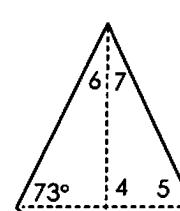
$$\boxed{= 47}$$

Name:

Date:

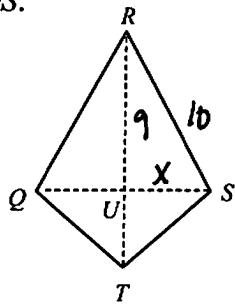
Topic:

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Main Ideas/Questions	Notes/Examples
Properties of KITES	<p>A kite is a quadrilateral with the following properties:</p>  <ul style="list-style-type: none"> <li>Exactly two pairs of consecutive congruent sides. (<math>\overline{AB} \cong \overline{AD}</math> and <math>\overline{BC} \cong \overline{DC}</math>)</li> <li>One pair of opposite angles are congruent. (<math>\angle ABC \cong \angle ADC</math>)</li> <li>Diagonals are perpendicular. (<math>\overline{AC} \perp \overline{BD}</math>)</li> </ul>
<b>Directions:</b> If each quadrilateral below is a kite, find the missing values.	
1.	 <p><math>m\angle B = \underline{116^\circ}</math> <math>m\angle D = \underline{116^\circ}</math></p>
2.	 <p><math>m\angle K = \underline{82^\circ}</math> <math>m\angle L = \underline{125^\circ}</math></p>
3.	 <p><math>m\angle PTQ = \underline{90^\circ}</math> <math>m\angle PQT = \underline{53^\circ}</math> <math>m\angle QRT = \underline{31^\circ}</math></p>
4.	 <p><math>m\angle GDE = \underline{118^\circ}</math> <math>m\angle DEH = \underline{31^\circ}</math> <math>m\angle DGH = \underline{31^\circ}</math></p>
5.	 <p><math>m\angle 1 = \underline{25^\circ}</math> <math>m\angle 2 = \underline{25^\circ}</math> <math>m\angle 3 = \underline{90^\circ}</math> <math>m\angle 4 = \underline{65^\circ}</math> <math>m\angle 5 = \underline{38^\circ}</math> <math>m\angle 6 = \underline{38^\circ}</math> <math>m\angle 7 = \underline{52^\circ}</math></p>
6.	 <p><math>m\angle 1 = \underline{46^\circ}</math> <math>m\angle 2 = \underline{44^\circ}</math> <math>m\angle 3 = \underline{44^\circ}</math> <math>m\angle 4 = \underline{90^\circ}</math> <math>m\angle 5 = \underline{73^\circ}</math> <math>m\angle 6 = \underline{17^\circ}</math> <math>m\angle 7 = \underline{17^\circ}</math></p>
7. If $WX = 14$ and $WR = 8$ , find $RZ$ .	$X^2 + 8^2 = 14^2$ $X^2 = 132$ $X = 11.5$ <p><math>RZ = 11.5</math></p>
8. If $AC = 38$ and $ED = 41$ , find $CD$ .	$19^2 + 41^2 = X^2$ $2042 = X^2$ $X = 45.2$ <p><math>CD = 45.2</math></p>

9. If  $RS = 10$  and  $RU = 9$ , find  $QS$ .

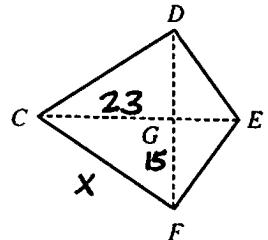
$$\begin{aligned} 9^2 + x^2 &= 10^2 \\ x^2 &= 19 \\ x &= 4.4 \end{aligned}$$



$$QS = 2(4.4) = 8.8$$

10. If  $GF = 15$  and  $CG = 23$ , find  $CD$ .

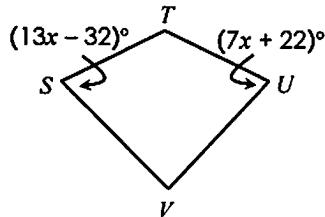
$$\begin{aligned} 15^2 + 23^2 &= x^2 \\ 754 &= x^2 \\ 27.5 &= x \end{aligned}$$



$$CD = 27.5$$

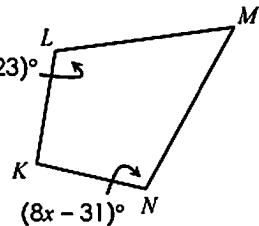
11. Solve for  $x$ .

$$\begin{aligned} 13x - 32 &= 7x + 22 \\ 6x &= 54 \\ x &= 9 \end{aligned}$$



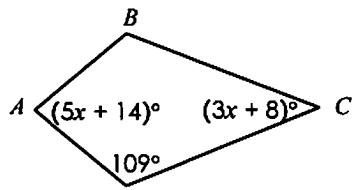
12. Find  $m\angle L$ .

$$\begin{aligned} 5x + 23 &= 8x - 31 \\ 54 &= 3x \\ 18 &= x \end{aligned}$$



$$\begin{aligned} m\angle L &= 5(18) + 23 \\ &= 113^\circ \end{aligned}$$

13. Solve for  $x$ .



$$5x + 14 + 3x + 8 + 2(109) = 360$$

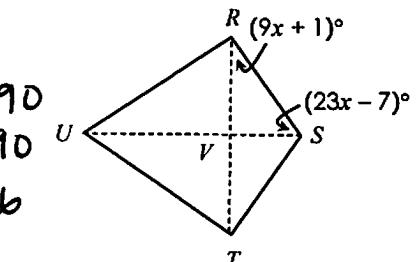
$$8x + 240 = 360$$

$$8x = 120$$

$$x = 15$$

14. Find  $m\angle STV$ .

$$\begin{aligned} 9x + 1 + 23x - 7 &= 90 \\ 32x - 6 &= 90 \\ 32x &= 96 \\ x &= 3 \end{aligned}$$



$$m\angle RV = 9(3) + 1 = 28^\circ$$

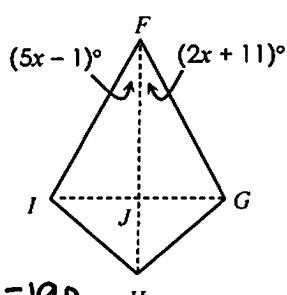
$$m\angle STV = 28^\circ$$

15. Find  $m\angle FGJ$ .

$$\begin{aligned} 5x - 1 &= 2x + 11 \\ 3x &= 12 \\ x &= 4 \end{aligned}$$

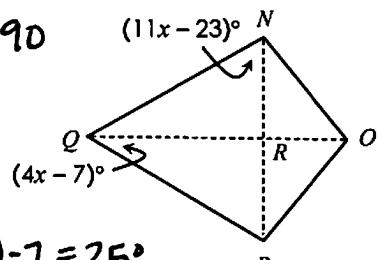
$$m\angle JFG = 2(4) + 11 = 19^\circ$$

$$m\angle FGI = 90 - 19 = 71^\circ$$



16. Find  $m\angle NQP$ .

$$\begin{aligned} 11x - 23 + 4x - 7 &= 90 \\ 15x &= 120 \\ x &= 8 \end{aligned}$$

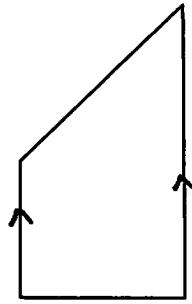


$$m\angle PQR = 4(8) - 7 = 25^\circ$$

$$m\angle NQP = 50^\circ$$

# QUADRILATERALS

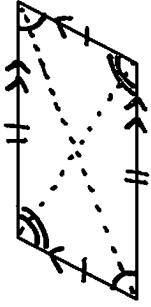
## TRAPEZOID



- Only ONE pair of opposite sides parallel (called bases).
- Consecutive angles are supplementary.

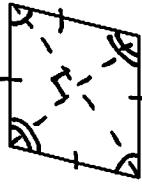
**Midsegment of a Trapezoid:**  
A midsegment of a trapezoid connects the midpoints of the legs. This segment is equal to the average of the two bases.

## PARALLELGRAM



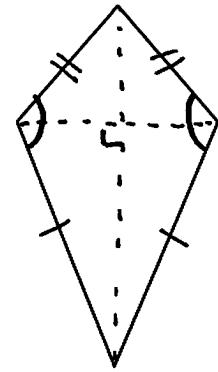
- Opposite sides parallel.
- Opposite sides congruent.
- Opposite angles congruent.
- Consecutive angles supplementary.
- Diagonals bisect each other.

## RHOMBUS



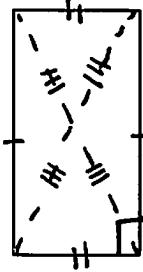
- Four congruent sides.
- Diagonals are perpendicular.
- Diagonals bisect opposite angles.

## KITE



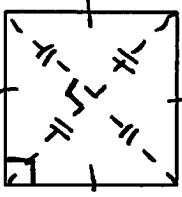
- Exactly two pairs of consecutive congruent sides.
- One pair of opposite angles are congruent.
- Diagonals are perpendicular.

## RECTANGLE

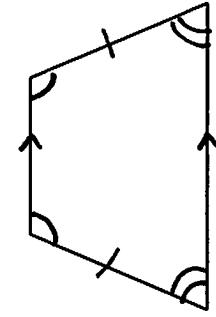


- Four right angles.
- Diagonals are congruent.

## SQUARE



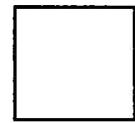
Squares have ALL the properties of parallelograms, rectangles, and rhombi!



- Non-parallel sides (legs) are congruent.
- Diagonals are congruent.
- Base angles are congruent.
- Opposite angles are supplementary.

## ISOSCELES TRAPEZOID

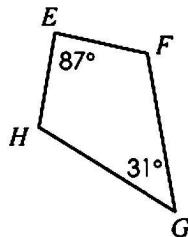
Name: \_\_\_\_\_

**Unit 8: Polygons & Quadrilaterals**

Date: \_\_\_\_\_ Per: \_\_\_\_\_

**Homework 8: Kites****\*\* This is a 2-page document! \*\*****Directions:** If each quadrilateral below is a kite, find the missing measures.

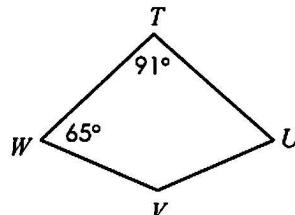
1.



$$m\angle F = 121^\circ$$

$$m\angle H = 121^\circ$$

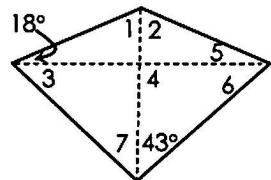
2.



$$m\angle U = 65^\circ$$

$$m\angle V = 139^\circ$$

3.



$$m\angle 1 = 72^\circ$$

$$m\angle 5 = 18^\circ$$

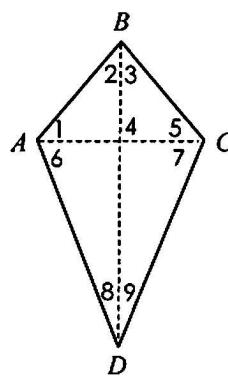
$$m\angle 2 = 72^\circ$$

$$m\angle 6 = 41^\circ$$

$$m\angle 3 = 41^\circ$$

$$m\angle 7 = 43^\circ$$

$$m\angle 4 = 90^\circ$$

4. Given:  $m\angle ABC = 70^\circ$  and  $m\angle ADC = 46^\circ$ .

$$m\angle 1 = 55^\circ$$

$$m\angle 2 = 35^\circ$$

$$m\angle 3 = 35^\circ$$

$$m\angle 4 = 90^\circ$$

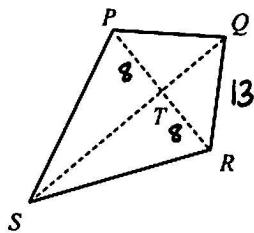
$$m\angle 5 = 55^\circ$$

$$m\angle 6 = 67^\circ$$

$$m\angle 7 = 67^\circ$$

$$m\angle 8 = 23^\circ$$

$$m\angle 9 = 23^\circ$$

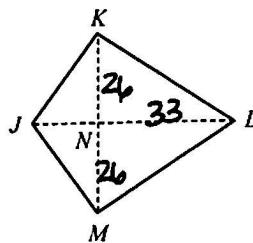
5. If  $QR = 13$  and  $PT = 8$ , find  $QT$ .

$$X^2 + 8^2 = 13^2$$

$$X^2 = 105$$

$$X = 10.2$$

$$\boxed{QT = 10.2}$$

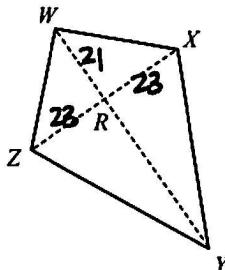
6. If  $KM = 52$  and  $NL = 33$ , find  $LM$ .

$$26^2 + 33^2 = X^2$$

$$1765 = X^2$$

$$X = 42$$

$$\boxed{LM = 42}$$

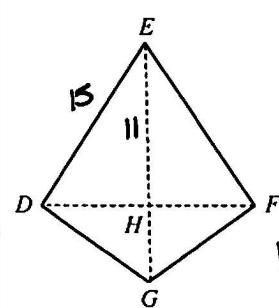
7. If  $XZ = 46$  and  $WR = 21$ , find  $WX$ .

$$21^2 + 23^2 = X^2$$

$$970 = X^2$$

$$31.1 = X$$

$$\boxed{WX = 31.1}$$

8. If  $DE = 15$  and  $EH = 11$ , find  $DF$ .

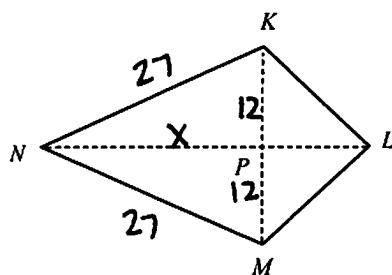
$$X^2 + 11^2 = 15^2$$

$$X^2 = 104$$

$$X = 10.2$$

$$DF = 2(10.2) \\ = \boxed{20.4}$$

9. If  $NK = 7x - 1$ ,  $NM = 10x - 13$ , and  $KM = 24$ , find  $NP$ .



$$7x - 1 = 10x - 13$$

$$12 = 3x$$

$$4 = x$$

$$x^2 + 12^2 = 27^2$$

$$x^2 = 585$$

$$x = 24.2$$

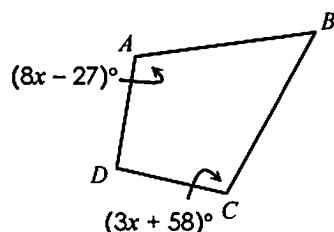
$$\boxed{NP = 24.2}$$

10. Solve for  $x$ .

$$8x - 27 = 3x + 58$$

$$5x = 85$$

$$\boxed{x = 17}$$

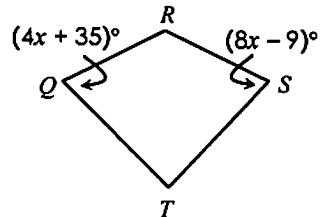


11. Find  $m\angle S$ .

$$8x - 9 = 4x + 35$$

$$4x = 44$$

$$x = 11$$



$$m\angle S = 8(11) - 9$$

$$\boxed{= 79^\circ}$$

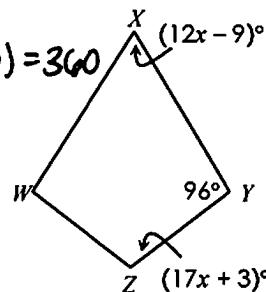
12. Solve for  $x$ .

$$12x - 9 + 17x + 3 + 2(96) = 360$$

$$29x + 186 = 360$$

$$29x = 174$$

$$\boxed{x = 6}$$

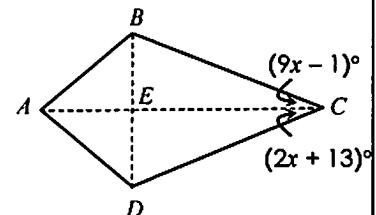


13. Find  $m\angle EDC$ .

$$9x - 1 = 2x + 13$$

$$7x = 14$$

$$x = 2$$



$$m\angle ECD = 2(2) + 13 = 17^\circ$$

$$\boxed{m\angle EDC = 73^\circ}$$

14. Find  $m\angle RST$ .

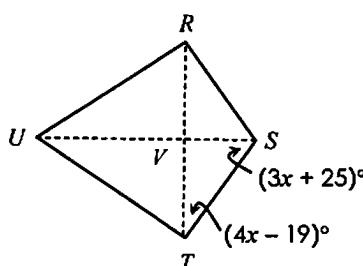
$$7x + 6 = 90$$

$$7x = 84$$

$$x = 12$$

$$m\angle TSV = 3(12) + 25 = 61^\circ$$

$$m\angle RST = 2(61) = \boxed{122^\circ}$$



15. Find  $m\angle HIF$ .

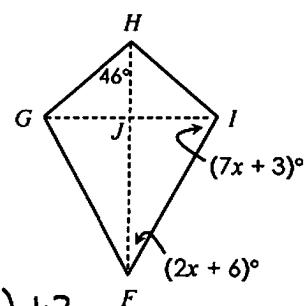
$$9x + 9 = 90$$

$$9x = 81$$

$$x = 9$$

$$m\angle HIF = 44 + 7(6) + 3$$

$$\boxed{= 110^\circ}$$



# Unit 8 Test Study Guide

## (Polygons & Quadrilaterals)

Name: \_\_\_\_\_

Date: \_\_\_\_\_ Per: \_\_\_\_\_

### Topic 1: Angles of Polygons

1. What is the formula for the sum of the interior angle measures of a polygon?

$$S = (n-2) \cdot 180$$

3. Find the sum of the interior angle measures of a 35-gon.

$$(35-2) \cdot 180 = 5440^\circ$$

5. If the sum of the interior angles of a polygon is  $3780^\circ$ , how many sides does it have?

$$\begin{aligned} 3780 &= (n-2) \cdot 180 \\ 21 &= n-2 \\ n &= 23 \end{aligned}$$

7. What is the sum of the exterior angle measures of any polygon?

$$360^\circ$$

9. If the exterior angle of a regular polygon measures  $24^\circ$ , how many sides does it have?

$$\frac{360}{24} = 15 \text{ sides}$$

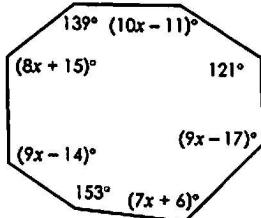
11. Solve for  $x$ .

$$(8-2) \cdot 180 = 1080$$

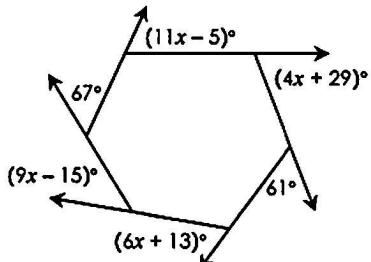
$$43x + 392 = 1080$$

$$43x = 688$$

$$x = 16$$



13. Solve for  $x$ .



2. What is the formula to find the measure of each interior angle of a regular polygon?

$$\frac{(n-2) \cdot 180}{n}$$

4. Six angles of a heptagon measure  $107^\circ$ ,  $139^\circ$ ,  $110^\circ$ ,  $145^\circ$ , and  $128^\circ$ . What is the measure of the seventh angle?

$$\begin{aligned} (7-2) \cdot 180 &= 900 \\ 900 - 760 &= 140^\circ \\ &= 900 \end{aligned}$$

6. What is the measure of each interior angle of a regular 18-gon?

$$\begin{aligned} (18-2) \cdot 180 &= 2880 \\ 2880 &= 180^\circ \\ 18 &= 160^\circ \end{aligned}$$

8. What is the measure of each exterior angle of a regular 30-gon?

$$\frac{360}{30} = 12^\circ$$

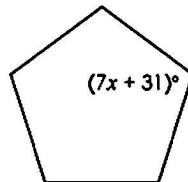
10. If the interior angle of a regular polygon is  $162^\circ$ , how many sides does it have?

$$\begin{aligned} \text{ext} &= 18^\circ \\ \frac{360}{18} &= 20 \text{ sides} \end{aligned}$$

12. If the figure is a regular polygon, solve for  $x$ .

$$\frac{(5-2) \cdot 180}{5} = 108$$

$$\begin{aligned} 7x + 31 &= 108 \\ 7x &= 77 \\ x &= 11 \end{aligned}$$



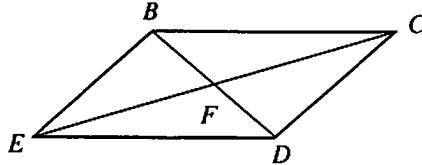
$$30x + 150 = 360$$

$$30x = 210$$

$$x = 7$$

### Topic 2: Parallelograms

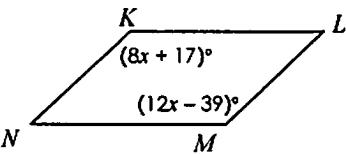
14. If  $EBCD$  is a parallelogram,  $EB = 16$ ,  $ED = 25$ ,  $BF = 11$ ,  $EC = 34$ ,  $m\angle BED = 55^\circ$ ,  $m\angle CDB = 67^\circ$ , and  $m\angle BCE = 24^\circ$ , find each missing measure.



$$\begin{array}{ll} BC = \underline{\quad 25 \quad} & m\angle EDC = \underline{\quad 125^\circ \quad} \\ BD = \underline{\quad 22 \quad} & m\angle EBD = \underline{\quad 67^\circ \quad} \\ FC = \underline{\quad 17 \quad} & m\angle BEC = \underline{\quad 31^\circ \quad} \\ CD = \underline{\quad 16 \quad} & m\angle DBC = \underline{\quad 58^\circ \quad} \end{array}$$

15. Find  $m\angle N$ .

$$\begin{aligned} 12x - 39 &= 8x + 17 \\ 4x &= 56 \\ x &= 14 \end{aligned}$$

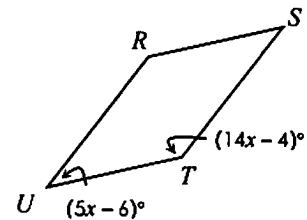


$$m\angle K: 8(14) + 17 = 129^\circ$$

$$m\angle N = 180 - 129 = \boxed{51^\circ}$$

16. Find  $m\angle R$ .

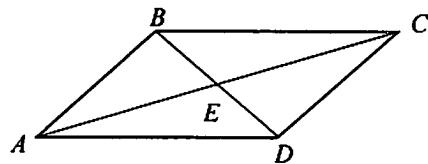
$$\begin{aligned} 19x - 10 &= 180 \\ 19x &= 190 \\ x &= 10 \end{aligned}$$



$$m\angle T = 14(10) - 4 = 136^\circ$$

$$m\angle R = \boxed{136^\circ}$$

17. In parallelogram  $ABCD$ , if  $ED = 7x - 13$  and  $BD = 16x - 38$ , find  $BD$ .

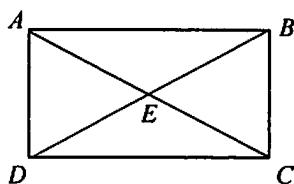


$$\begin{aligned} 2(7x - 13) &= 16x - 38 \\ 14x - 26 &= 16x - 38 \\ 12 &= 2x \\ x &= 6 \end{aligned}$$

$$\begin{aligned} BD &= 16(6) - 38 \\ &= \boxed{58} \end{aligned}$$

### Topic 3: Rectangles

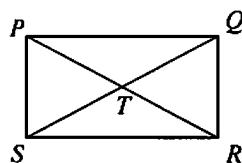
18. If  $ABCD$  is a rectangle,  $AD = 9$ ,  $AC = 22$ , and  $m\angle BCA = 66^\circ$ , find each missing measure.



$$\begin{aligned} AB: x^2 + 9^2 &= 22^2 \\ x^2 &= 403 \\ x &= 20.1 \end{aligned}$$

$$\begin{array}{ll} BC = \underline{\quad 9 \quad} & m\angle ADC = \underline{\quad 90^\circ \quad} \\ AB = \underline{\quad 20.1 \quad} & m\angle BAC = \underline{\quad 24^\circ \quad} \\ BD = \underline{\quad 22 \quad} & m\angle CDB = \underline{\quad 24^\circ \quad} \\ EC = \underline{\quad 11 \quad} & m\angle AEB = \underline{\quad 132^\circ \quad} \end{array}$$

19. If  $PQRS$  is a rectangle,  $PR = 9x + 1$ , and  $QS = 13x - 11$ , find  $TR$ .

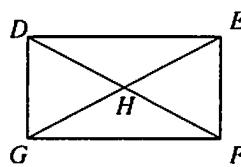


$$\begin{aligned} 13x - 11 &= 9x + 1 \\ 4x &= 12 \\ x &= 3 \end{aligned}$$

$$PR = 9(3) + 1 = 28$$

$$TR = \boxed{14}$$

20. If  $DEFG$  is a rectangle,  $m\angle DEG = (4x - 5)^\circ$ , and  $m\angle FGE = (6x - 21)^\circ$ , find  $m\angle DGE$ .

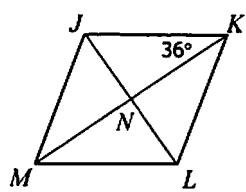


$$\begin{aligned} 6x - 21 &= 4x - 5 \\ 2x &= 16 \\ x &= 8 \end{aligned}$$

$$\begin{aligned} m\angle FGE &= 6(8) - 21 \\ &= 27^\circ \\ m\angle DGE &= \boxed{63^\circ} \end{aligned}$$

### Topic 4: Rhombi & Squares

21. If  $JKLM$  is a rhombus, find each angle.

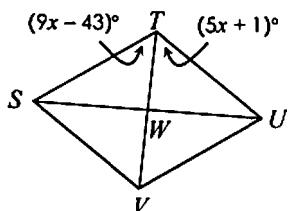


$$\begin{aligned}m\angle JKL &= 72^\circ \\ m\angle MLK &= 108^\circ \\ m\angle JMK &= 36^\circ \\ m\angle MJL &= 54^\circ \\ m\angle KNL &= 90^\circ\end{aligned}$$

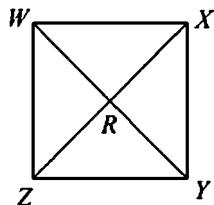
23. If  $STUV$  is a rhombus, find  $m\angle SVU$ .

$$\begin{aligned}9x - 43 &= 5x + 1 \\ 4x &= 44 \\ x &= 11\end{aligned}$$

$$m\angle SVU = 112^\circ$$



25. If  $WXYZ$  is a square, find each angle.



$$\begin{aligned}m\angle WXY &= 90^\circ \\ m\angle XZY &= 45^\circ \\ m\angle YXZ &= 45^\circ \\ m\angle WRZ &= 90^\circ \\ m\angle XWY &= 45^\circ\end{aligned}$$

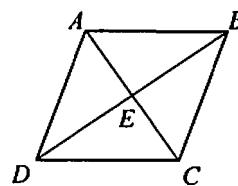
22. Using  $JKLM$  from the previous question, if  $MN = 20$  and  $JL = 26$ , find  $JK$ .

$$\begin{aligned}13^2 + 20^2 &= x^2 \\ 569 &= x^2 \\ x &= 23.9\end{aligned}$$

$$JK = 23.9$$

24. If  $ABCD$  is a rhombus,  $AD = 4x + 2$ ,  $DC = 7x - 13$ , and  $BD = 34$ , find  $AE$ .

$$\begin{aligned}7x - 13 &= 4x + 2 \\ 3x &= 15 \\ x &= 5\end{aligned}$$



$$\begin{aligned}17^2 + x^2 &= 22^2 \\ x^2 &= 195 \\ x &= 14\end{aligned}$$

$$AE = 14$$

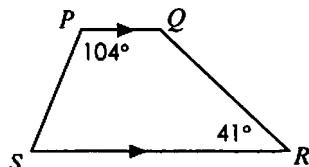
26. Using  $WXYZ$  from the previous question, if  $WY = 32$ , find  $XY$ .

$$\begin{aligned}16^2 + 16^2 &= x^2 \\ 512 &= x^2 \\ x &= 22.6\end{aligned}$$

$$XY = 22.6$$

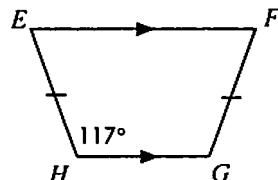
### Topic 5: Trapezoids

27. Find each measure.



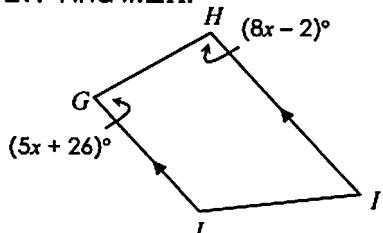
$$\begin{aligned}m\angle Q &= 139^\circ \\ m\angle S &= 76^\circ\end{aligned}$$

28. Find each measure.



$$\begin{aligned}m\angle E &= 63^\circ \\ m\angle F &= 63^\circ \\ m\angle G &= 117^\circ\end{aligned}$$

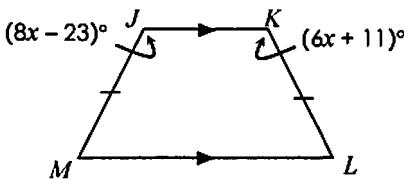
29. Find  $m\angle H$ .



$$\begin{aligned}13x + 24 &= 180 \\ 13x &= 156 \\ x &= 12\end{aligned}$$

$$m\angle H = 94^\circ$$

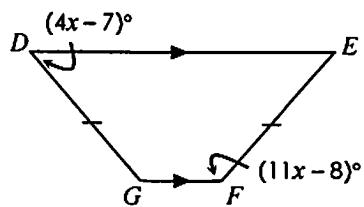
30. Find  $m\angle M$ .



$$\begin{aligned}8x - 23 &= 6x + 11 \\ 2x &= 34 \\ x &= 17\end{aligned}$$

$$\begin{aligned}m\angle J &= 113^\circ \\ m\angle M &= 67^\circ\end{aligned}$$

31. Find  $m\angle G$ .



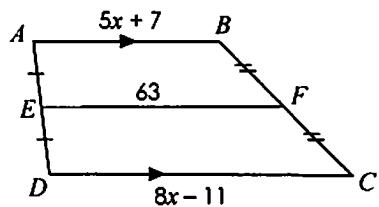
$$4x - 7 + 11x - 8 = 180$$

$$15x = 195$$

$$x = 13$$

$$m\angle F = 135^\circ ; \boxed{m\angle G = 135^\circ}$$

33. Find  $AB$ .



$$63 = \frac{5x + 7 + 8x - 11}{2}$$

$$126 = 13x - 4$$

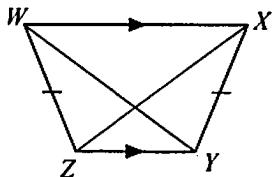
$$130 = 13x$$

$$x = 10$$

$$AB = 5(10) + 7$$

$$= \boxed{57}$$

32. If  $WY = 15x - 2$  and  $XZ = 9x + 10$ , find  $WY$ .



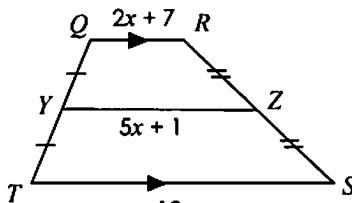
$$15x - 2 = 9x + 10$$

$$6x = 12$$

$$x = 2$$

$$WY = 15(2) - 2 = \boxed{28}$$

34. Find  $YZ$ .



$$5x + 1 = \frac{2x + 7 + 19}{2}$$

$$10x + 2 = 2x + 26$$

$$8x = 24$$

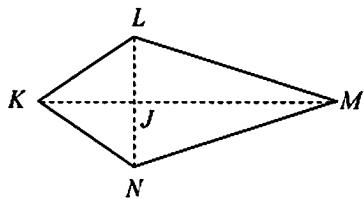
$$x = 3$$

$$YZ = 5(3) + 1$$

$$= \boxed{16}$$

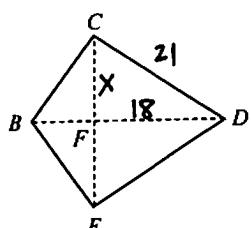
### Topic 6: Kites

35. If  $KLMN$  is a kite,  $m\angle LMN = 36^\circ$  and  $m\angle KNJ = 54^\circ$ , find each measure.



$$\begin{aligned} m\angle KLN &= 54^\circ & m\angle JKN &= 36^\circ \\ m\angle LKN &= 72^\circ & m\angle NMJ &= 18^\circ \\ m\angle KNM &= 126^\circ & m\angle JLM &= 72^\circ \\ m\angle LJM &= 90^\circ & m\angle KLM &= 126^\circ \end{aligned}$$

36. If  $BCDE$  is a kite,  $CD = 21$  and  $DF = 18$ , find  $CE$ .



$$x^2 + 18^2 = 21^2$$

$$x^2 = 117$$

$$x = 10.8$$

$$CE = 2(10.8)$$

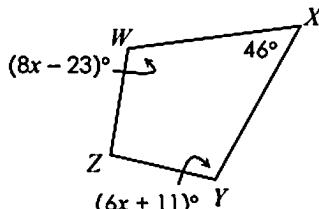
$$= \boxed{21.6}$$

37. If  $WXYZ$  is a kite, find  $m\angle Z$ .

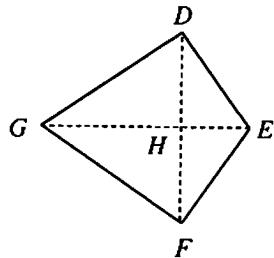
$$\begin{aligned} 8x - 23 &= 6x + 11 \\ 2x &= 34 \\ x &= 17 \end{aligned}$$

$$\begin{aligned} m\angle W &= 8(17) - 23 \\ &= 113^\circ \end{aligned}$$

$$\begin{aligned} m\angle Z &= 360 - 272 \\ &= \boxed{88^\circ} \end{aligned}$$



38. If  $DEFG$  is a kite,  $m\angle DEF = (12x - 16)^\circ$ ,  $m\angle EFH = (3x - 1)^\circ$  and  $m\angle DGF = 74^\circ$ , find  $m\angle GFE$ .



$$6x - 8 + 3x - 1 + 90 = 180$$

$$9x + 81 = 180$$

$$9x = 99$$

$$x = 11$$

$$\begin{aligned} m\angle DEF &= 12(11) - 16 \\ &= 116^\circ \end{aligned}$$

$$m\angle GFE = \frac{360 - 190}{2}$$

$$= 85^\circ$$

### Topic 7: Quadrilaterals in the Coordinate Plane

Use the distance and slope formulas to justify your answers to questions 39-40.

39. Determine whether  $WXYZ$  given  $W(0, 8)$ ,  $X(6, 10)$ ,  $Y(-1, -1)$ ,  $Z(-7, -3)$  is a parallelogram.

$$m(\overline{WX}) = \frac{10-8}{6-0} = \frac{2}{6} = \frac{1}{3}$$

$$\left. \begin{array}{l} \\ \end{array} \right\} \overline{WX} \parallel \overline{YZ}$$

$$m(\overline{YZ}) = \frac{-3+1}{-7+1} = \frac{-2}{-6} = \frac{1}{3}$$

Yes; parallelogram!

$$m(\overline{XY}) = \frac{-1-10}{-1-6} = \frac{-11}{-7} = \frac{11}{7}$$

$$\left. \begin{array}{l} \\ \end{array} \right\} \overline{XY} \parallel \overline{WZ}$$

$$m(\overline{WZ}) = \frac{-3-8}{-7-0} = \frac{-11}{-7} = \frac{11}{7}$$

40. Determine the most precise classification for quadrilateral  $ABCD$  (parallelogram, rectangle, rhombus, or square) given  $A(3, -4)$ ,  $B(10, -2)$ ,  $C(8, -9)$ ,  $D(1, -11)$ .

$$AB: d = \sqrt{(3-10)^2 + (-4+2)^2} = \sqrt{49+4} = \sqrt{53}$$

$$CD: d = \sqrt{(8-1)^2 + (-9+11)^2} = \sqrt{49+4} = \sqrt{53}$$

$$BC: d = \sqrt{(10-8)^2 + (-2+9)^2} = \sqrt{4+49} = \sqrt{53}$$

$$AD: d = \sqrt{(3-1)^2 + (-4+11)^2} = \sqrt{4+49} = \sqrt{53}$$

$$AC: d = \sqrt{(3-8)^2 + (-4+9)^2} = \sqrt{25+25} = \sqrt{50}$$

$$BD: d = \sqrt{(10-1)^2 + (-2+11)^2} = \sqrt{81+81} = \sqrt{162}$$

Sq / Rhm

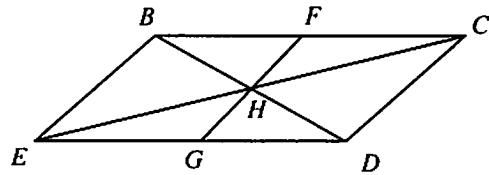
Rhm

Rhombus

### Topic 8: Parallelogram Proofs

41. Given:  $BCDE$  is a parallelogram

Prove:  $\overline{EG} \cong \overline{CF}$

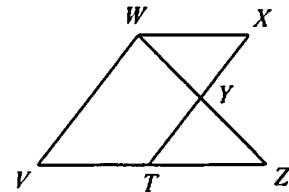


Statements	Reasons
1. $BCDE$ is a parallelogram	1. given
2. $\overline{BC} \parallel \overline{ED}$	2. Def of parallelogram
3. $\angle BCE \cong \angle DEC$	3. Alt. Interior Angles
4. $\angle CFG \cong \angle EGF$	4. Alt. Interior Angles
5. $\overline{EH} \cong \overline{CH}$	5. Diag of a p'gram bisect each other.
6. $\triangle EGH \cong \triangle CFH$	6. AAS $\triangle$ Congruence
7. $\overline{EG} \cong \overline{CF}$	7. CPCTC

42. Given:  $\triangle TYZ \cong \triangle XYW$

T is the midpoint of  $\overline{VZ}$

Prove:  $VWXT$  is a parallelogram



Statements	Reasons
1. $\triangle TYZ \cong \triangle XYW$	1. given
2. $\overline{ZY} \cong \overline{WX}$	2. CPCTC
3. T is the midpoint of $\overline{VZ}$	3. given
4. $\overline{VT} \cong \overline{ZT}$	4. Def of midpoint
5. $\overline{VT} \cong \overline{WX}$	5. Transitive Property
6. $\angle ZTY \cong \angle WXY$	6. CPCTC
7. $\overline{VT} \parallel \overline{WX}$	7. Alt. Interior Angles
8. $VWXT$ is a parallelogram.	8. One pair of opp sides are parallel & congruent.

Name: \_\_\_\_\_

## Unit 8 Test

Date: \_\_\_\_\_ Per: \_\_\_\_\_

## Polygons &amp; Quadrilaterals

1. What is the sum of the measures of the interior angles of a 27-gon?

$$(27-2) \cdot 180$$

- A.  $4860^\circ$   
B.  $4500^\circ$   
C.  $5220^\circ$   
D.  $166.7^\circ$

B

2. If the sum of the interior angles of a polygon is  $2340^\circ$ , how many sides does the polygon have?

$$(n-2) \cdot 180 = 2340$$

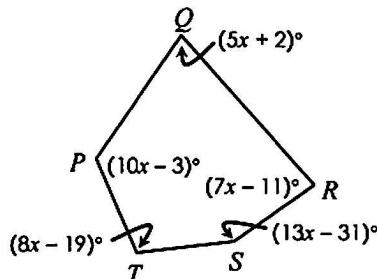
$$n-2 = 13$$

- A. 11 sides  
B. 13 sides  
C. 15 sides  
D. 16 sides

C

3. Find  $m\angle S$ .

$$(5-2) \cdot 180 = 540$$



$$43x - 62 = 540$$

$$43x = 602$$

$$x = 14$$

$$m\angle S = 13(14) - 31$$

$$= 151^\circ$$

$$m\angle S = 151^\circ$$

4. A regular hexagon is shown below. Find the value of  $x$ .

$$11x + 21 = 120$$

$$11x = 99$$

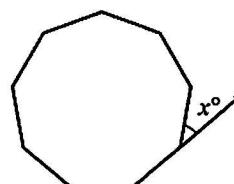
$$x = 9$$



$$x = 9$$

5. If the polygon shown below is a regular nonagon, what is the value of  $x$ ?

$$\frac{360}{9} = 40$$

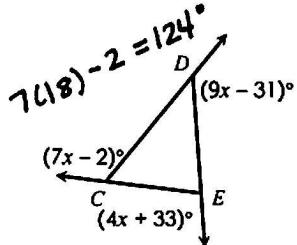


$$x = 40$$

6. Find  $m\angle DCE$ .

$$20x = 360$$

$$x = 18$$



$$m\angle DCE = 56^\circ$$

7. If each interior angle of a regular polygon measures  $168^\circ$ , how many sides does the polygon have?

$$\text{ext} = 12^\circ$$

$$\frac{360}{12}$$

- A. 12 sides  
B. 30 sides  
C. 25 sides  
D. 15 sides

$$B$$

8. Which of the following properties is always true for a parallelogram?

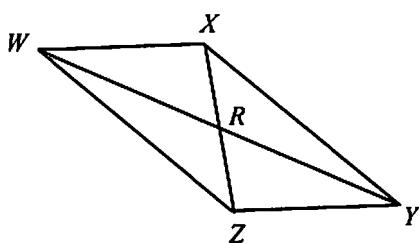
- A. Diagonals bisect each other.  
B. Diagonals are perpendicular.  
C. Diagonals are congruent.  
D. Diagonals bisect opposite angles.

$$A$$

9. Which of the following quadrilaterals always have diagonals that are congruent? Check all that apply.

- Parallelograms  
 Rectangles  
 Rhombi  
 Squares  
 Isosceles Trapezoids

**Use parallelogram  $WXYZ$  for questions 10 and 11.**



10. If  $m\angle XYZ = 68^\circ$  and  $m\angle WXZ = 71^\circ$ , find  $m\angle WZX$ .

$$m\angle WZX = 41^\circ$$

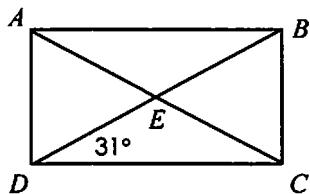
11. If  $XZ = 8x - 18$  and  $RZ = 2x + 5$ , find  $XR$ .

$$\begin{aligned} 8x - 18 &= 2(2x + 5) \\ 8x - 18 &= 4x + 10 \\ 4x &= 28 \\ x &= 7 \end{aligned}$$

$$\begin{aligned} RZ &= 2(7) + 5 \\ &= 19 \end{aligned}$$

$$XR = 19$$

**Use rectangle  $ABCD$  for questions 12-14.**



12. If  $EC = 13$ , find  $BD$ .

$$BD = 26$$

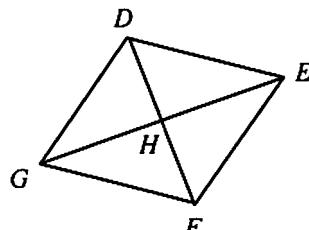
13. Find  $m\angle ADB$ .

$$m\angle ADB = 59^\circ$$

14. Find  $m\angle DEC$ .

$$m\angle DEC = 118^\circ$$

**Use rhombus  $DEFG$  for questions 15 and 16.**



15. If  $GE = 42$  and  $DH = 16$ , find  $GF$ .

$$\begin{aligned} 21^2 + 16^2 &= x^2 \\ 697 &= x^2 \\ x &= 26.4 \end{aligned}$$

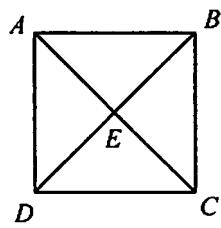
$$GF = 26.4$$

16. If  $EF = 13$  and  $DF = 18$ , find  $EH$ .

$$\begin{aligned} x^2 + 9^2 &= 13^2 \\ x^2 &= 88 \\ x &= 9.4 \end{aligned}$$

$$EH = 9.4$$

**Use square  $ABCD$  for questions 17 and 18.**



17. If  $AC = 26$ , find  $BC$ .

$$\begin{aligned} 13^2 + 13^2 &= x^2 \\ 338 &= x^2 \\ x &= 18.4 \end{aligned}$$

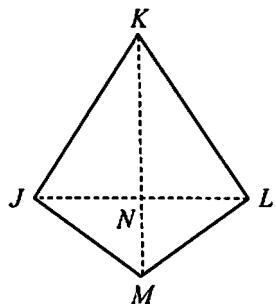
$$BC = 18.4$$

18. If  $m\angle ACB = (11x - 32)^\circ$ , find the value of  $x$ .

$$\begin{aligned} 11x - 32 &= 45 \\ 11x &= 77 \\ x &= 7 \end{aligned}$$

$$x = 7$$

Use kite  $JKLM$  for questions 19 and 20.



19. If  $m\angle JKN = 28^\circ$  and  $m\angle KLM = 103^\circ$ , find  $m\angle JML$ .

$$m\angle JML = 98^\circ$$

20. If  $JL = 18$ ,  $NK = 12$ , and  $ML = 10$ , find the perimeter of  $JKLM$ .

- A. 42  
B. 45  
C. 50  
D. 56

$$9^2 + 12^2 = c^2$$

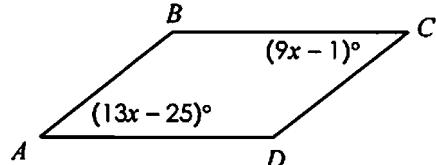
$$225 = c^2$$

$$c = 15$$

$$\text{Perimeter} = 2(15) + 2(10)$$

C

21. If  $ABCD$  is a parallelogram, find  $m\angle D$ .



$$13x - 25 = 9x - 1$$

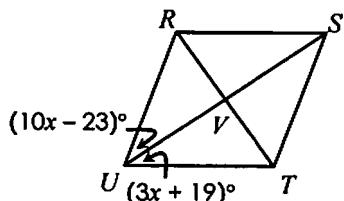
$$4x = 24$$

$$x = 6$$

$$m\angle C = 9(6) - 1 = 53^\circ$$

$$m\angle D = 127^\circ$$

23. If  $RSTU$  is a rhombus, find  $m\angle UTS$ .



$$10x - 23 = 3x + 19$$

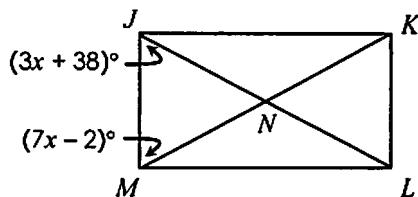
$$7x = 42$$

$$x = 6$$

$$\begin{aligned} m\angle VUT &= \\ &3(6) + 19 \\ &= 37^\circ \end{aligned}$$

$$m\angle UTS = 106^\circ$$

22. If  $JKLM$  is a rectangle, find  $m\angle NML$ .



$$7x - 2 = 3x + 38$$

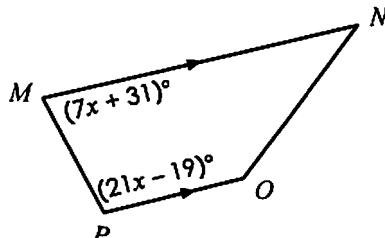
$$4x = 40$$

$$x = 10$$

$$m\angle NML = 7(10) - 2 = 68^\circ$$

$$m\angle NML = 22^\circ$$

24. Find  $m\angle P$ .



$$28x + 12 = 180$$

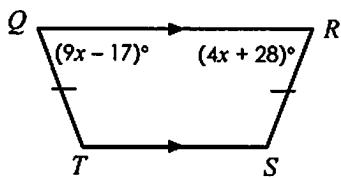
$$28x = 168$$

$$x = 6$$

$$\begin{aligned} m\angle P &= 21(6) - 19 \\ &= 107^\circ \end{aligned}$$

$$m\angle P = 107^\circ$$

- 25.** Find  $m\angle T$ .

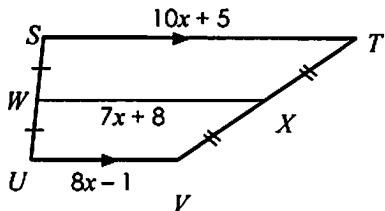


$$\begin{aligned} 9x - 17 &= 4x + 28 \\ 5x &= 45 \\ x &= 9 \end{aligned}$$

$$m\angle Q = 9(9) - 17 \\ \qquad \qquad \qquad = 64^\circ$$

$$m\angle T = 116^\circ$$

- 27.** Find  $WX$ .



$$7x + 8 = \frac{10x + 5 + 8x - 1}{2}$$

$$14x + 16 = 18x + 4$$

$$12 = 4x$$

$$x = 3$$

$$wx = 7(3) + 8 \\ = 29$$

$$x = 3$$

$$WX = 29$$

29. The vertices of quadrilateral  $PQRS$  are given below. Use the distance and/or slope formulas to determine if  $PQRS$  is a parallelogram. Use the diagram as a guide.

- $$P(-6, 4), Q(-2, 7), R(-1, 0), S(-5, -3)$$

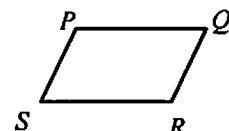
$$m(\overline{PQ}) = \frac{7-4}{-2+1} = \frac{3}{4}$$

$$m(\overline{RS}) = \frac{-3 - 0}{-5 + 1} = \frac{-3}{-4} = \frac{3}{4}$$

$$m(\overline{QR}) = \frac{0-7}{-1+2} = \frac{-7}{1} = -7$$

$$m(\overrightarrow{PS}) = \frac{-3-4}{-5+1} = \frac{-7}{1} = -7$$

$$\} \overline{PQ} \parallel \overline{RS}$$



- $PQRS$  is a parallelogram  
  $PQRS$  is not a parallelogram

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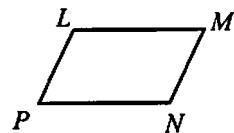
30. The vertices of quadrilateral  $LMNP$  are  $L(-1, 7)$ ,  $M(4, 9)$ ,  $N(8, -1)$ , and  $P(3, -3)$ . Using the distance formula, determine the most precise classification of  $LMNP$ : parallelogram, rectangle, rhombus, or square. Use the diagram as a guide.

$$LM = \sqrt{(4+1)^2 + (9-7)^2} = \sqrt{25+4} = \sqrt{29}$$

$$MN = \sqrt{(8-4)^2 + (-1-9)^2} = \sqrt{16+100} = \sqrt{116} = 2\sqrt{29}$$

$$NP = \sqrt{(3-8)^2 + (-3+1)^2} = \sqrt{25+4} = \sqrt{29}$$

$$LP = \sqrt{(3+1)^2 + (-3-7)^2} = \sqrt{16+100} = \sqrt{116} = 2\sqrt{29}$$



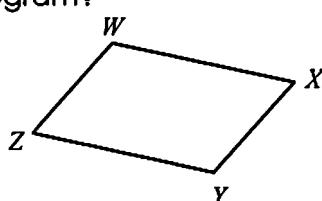
Rect or  
P'gram

$$LN = \sqrt{(8+1)^2 + (-1-7)^2} = \sqrt{81+64} = \sqrt{145}$$

$$MP = \sqrt{(3-4)^2 + (-3-9)^2} = \sqrt{1+144} = \sqrt{145}$$

$LMNP$  is a Rectangle

31.  $WXYZ$  is a quadrilateral with  $W$  located at  $(-5, 2)$  and  $X$  located at  $(3, 0)$ . What must be the slope of  $\overline{ZY}$  in order for  $WXYZ$  to be a parallelogram?



A. 4

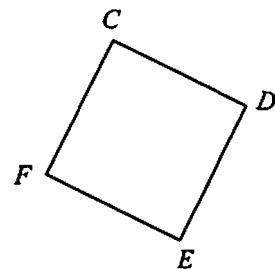
C.  $\frac{1}{4}$

B. -4

D.  $-\frac{1}{4}$

 D

32. Rhombus  $CDEF$  is shown below. If the slope of  $\overline{FC}$  is  $\frac{5}{2}$ , what must be the slope of  $\overline{CD}$  in order for  $CDEF$  to be a square?



A.  $\frac{2}{5}$

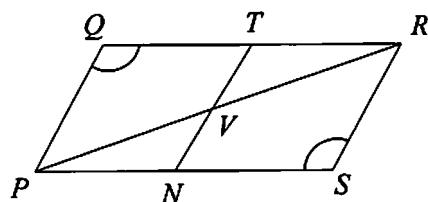
C.  $\frac{5}{2}$

B.  $-\frac{2}{5}$

D.  $-\frac{5}{2}$

 B

33. In addition to the information given in the drawing, which statement is sufficient to prove  $PQRS$  is a parallelogram?



A.  $\overline{QR} \cong \overline{SP}$

B.  $\overline{QP} \cong \overline{SR}$

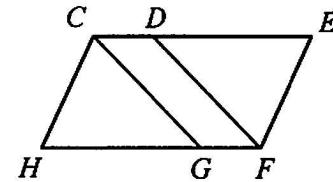
C.  $V$  is the midpoint of  $\overline{PR}$

D.  $\angle QPR \cong \angle SRP$

 D

34. Given:  $CEFH$  is a parallelogram,  $\triangle CGH \cong \triangle FDE$

Prove:  $CDFG$  is a parallelogram

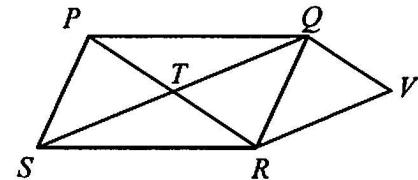


Statements	Reasons
1. $CEFH$ is a parallelogram	1. given
2. $\overline{CE} \parallel \overline{HF}$	2. Def. of parallelogram
3. $\angle GFD \cong \angle EDF$	3. Alt. Interior Angles
4. $\triangle CGH \cong \triangle FDE$	4. Given
5. $\angle EDF \cong \angle HGC$	5. CPCTC
6. $\overline{CG} \cong \overline{FD}$	6. CPCTC
7. $\angle GFD \cong \angle HGC$	7. Transitive Property
8. $\overline{CG} \parallel \overline{FD}$	8. Corresponding Angles Converse
9. $CDFG$ is a parallelogram	9. Def of Parallelogram

35. Given:  $PQRS$  is a parallelogram,  $\angle PSQ \cong \angle VRQ$

$$\angle RTQ \cong \angle QVR$$

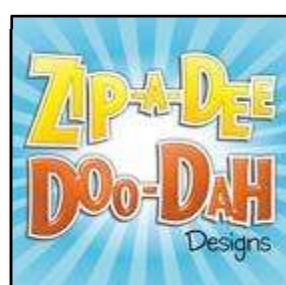
Prove:  $TQVR$  is a parallelogram



Statements	Reasons
1. $PQRS$ is a parallelogram	1. given
2. $\overline{PS} \parallel \overline{QR}$	2. Def of parallelogram
3. $\angle RQT \cong \angle PSQ$	3. Alt. Interior Angles
4. $\angle PSQ \cong \angle VRQ$	4. given
5. $\angle RQT \cong \angle VRQ$	5. Transitive Property
6. $\angle RTQ \cong \angle QVR$	6. Given
7. $\overline{QR} \cong \overline{QR}$	7. Reflexive Property
8. $\triangle RTQ \cong \triangle QVR$	8. AAS $\Delta$ Congruence
9. $\overline{TQ} \cong \overline{VR}$ ; $\overline{TR} \cong \overline{VQ}$	9. CPCTC
10. $TQVR$ is a parallelogram.	10. Opp. sides are congruent

# CREDITS

I use clipart and fonts in my products by:



Art with Jenny K



Many thanks to these talented artists!